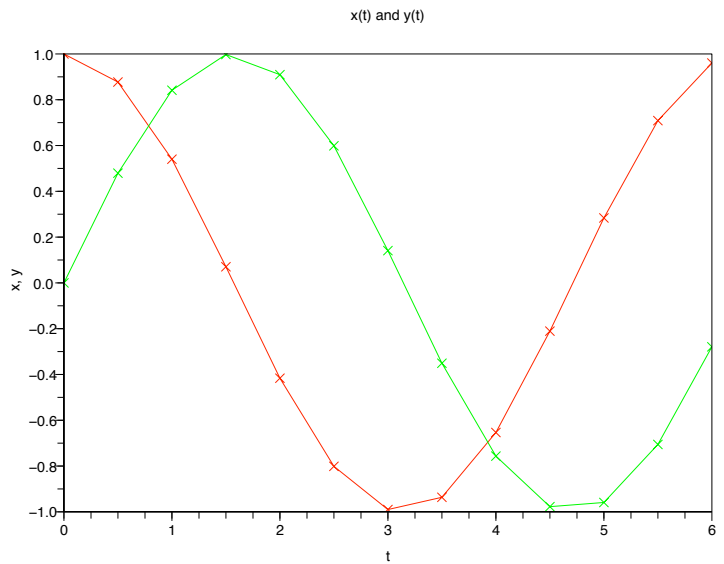
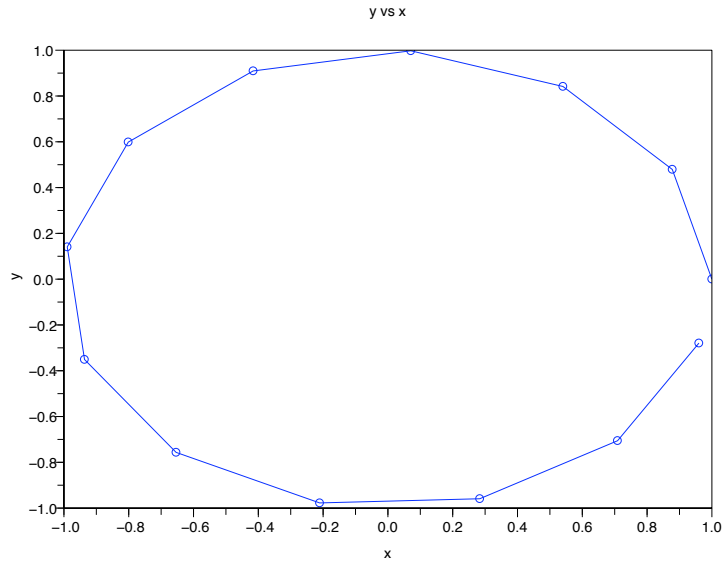


# GOING AROUND IN CIRCLES SAMPLE LAB WRITE UP AND LAB 9

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ABSTRACT. This is a sample lab write up



## INTRODUCTION

Our problem is to use Scilab to numerically study a sample ODE. Scilab is a command line programming language whose fundamental data structure is a matrix. Even for our simplest ODE's the matrix structure will become important.

Our sample system is the ODE

$$\frac{dx}{dt} = -y$$

$$\frac{dy}{dt} = x$$

and we will be interested in the initial value problem with  $x(0) = 1$  and  $y(0) = 0$ . The initial problem has an analytic solution given by

$$x(t) = \cos(t)$$

$$y(t) = \sin(t)$$

and we want to see plots of both  $x$  and  $y$  versus time  $t$  to see how these change with time. Also we want a phase diagram, the plot of the parametric equations  $(x(t), y(t))$ .

We convert to a matrix equation. We think of  $w(t) = \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix}$  as the column vector  $= \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$  and so the entry  $w_{1,1} = x$  and  $w_{2,1} = y$ . We enter our ODE system which is now

$$w'(t) = \begin{bmatrix} w'_1(t) \\ w'_2(t) \end{bmatrix} = \begin{bmatrix} -w_2(t) \\ w_1(t) \end{bmatrix} \quad w(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

into scilab using the function  $f(t, w)$ . We are defining the output as a column vector, the entry  $[a; b]$  translates to  $\begin{bmatrix} a \\ b \end{bmatrix}$  the semi-colon acting as a row separator. (The name `wdot` for  $w'(t)$  is a physics convention.)

```
-->function wdot = f(t, w)
-->wdot=[-w(2,1); w(1,1)];
-->endfunction;
```

Next we need a step size and a range. We start with

```
-->t=0:0.5:2*%pi;
```

This gives a step size of  $\Delta t = 0.5$ , and a range from 0 to  $2\pi$ . The `%pi` is Scilab's  $\pi$ . Actually since  $2\pi$  is not  $0 + 0.5n$ ,  $t$  stops at 6 that is  $t$  is the row vector

```
0 0.5 1 1.5 2 ... 6
```

so the colon operator stops before crossing the ending value.

We are ready to solve the ode. I always forget what order the parameters go into the ode function. But one can start up the help browser and search for ode. It is `ode(w0, t0, t, f)` for us it's

```
-->w=ode([1;0],0,t,f);
```

because our initial value is also a vector. The  $x(t)$  values are the first row of  $w$  or  $w(1, :)$  in scilab notation. Similarly,  $y(t)$  values are in  $w(2, :)$ . We can see the values best by

```

-->z=[t;w];
-->z'
ans =

    0.    1.    0.
    0.5  0.8775826  0.4794255
    1.    0.5403023  0.8414710
    1.5  0.0707372  0.9974950
    2.   - 0.4161468  0.9092974
    2.5 - 0.8011436  0.5984722
    3.   - 0.9899925  0.1411201
    3.5 - 0.9364567 - 0.3507832
    4.   - 0.6536437 - 0.7568025
    4.5 - 0.2107958 - 0.9775300
    5.    0.2836621 - 0.9589242
    5.5  0.7086697 - 0.7055403
    6.    0.9601700 - 0.2794155

```

The prime transposes the matrix  $z$ . The first row of  $z$  (first column of  $z'$ ) is  $t$ , the second row (column) is  $x$  and the last row (column) is  $y$ . If there is a lot of data, one can save directly to a file or use the command lines(0) and cut and paste.

For us the big bonus is to use scilab to plot these values. We got two plots, the phase plot in the file b1.pdf which plots ( $y$  vs  $x$ ) and then the plots of  $x$  and  $y$  versus time  $t$  in b2.eps.

```

-->plot(w(1,:),w(2:),'bo-');

-->xtitle("y vs x", "x", "y");

-->xs2eps(0,'b1.eps');

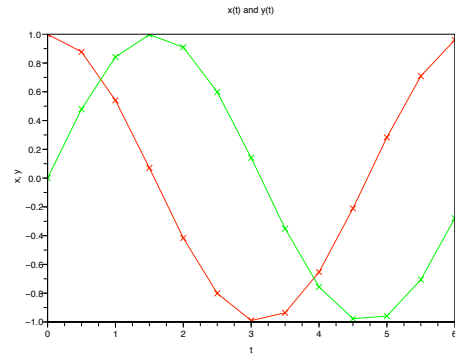
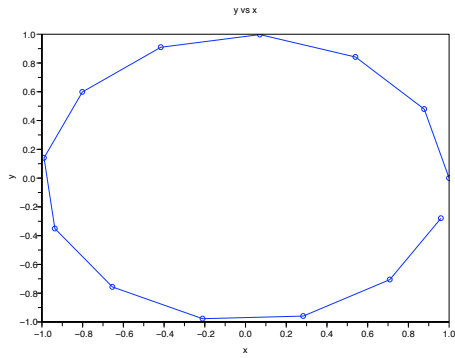
-->clf();

-->plot(t,w(1:),'rx-', t,w(2:), 'gx-');

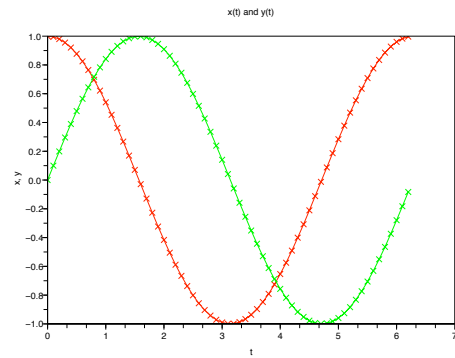
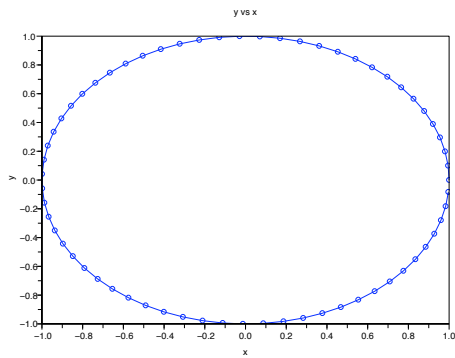
-->xtitle("x(t) and y(t)", "t", "x, y");

-->xs2eps(0,'b2.eps');

```



We repeated the assignment with  $\Delta t = 0.1$  and obtained the nicer:



YOUR LAB 9 ASSIGNMENT

This is the SIR model of a biological epidemic:

- In a total population of  $n$  individuals there are, at any time  $t$ ,
- $y(t)$  infectious carriers of a contagious disease,
- $x(t)$  members of the population who are susceptible to the disease, and
- $z(t)$  individuals who are recovered and immune.
- It is clear  $x(t) + y(t) + z(t) = n$  for all  $t \geq 0$ .

The following systems of equations model the epidemic

$$\begin{aligned} \frac{dx}{dt} &= -\lambda x(t)y(t) \\ \frac{dy}{dt} &= \lambda x(t)y(t) - \mu y(t) \\ \frac{dz}{dt} &= \mu y(t) \end{aligned}$$

- (1) Using the parameters  $t$  is time in days,  $\lambda = 0.001$ ,  $\mu = 1/14$ ,  $n = 1000$  and the initial conditions  $x(0) = 900$ ,  $y(0) = 10$ ,  $z(0) = 90$  and using scilab to compute and plot  $x(t)$ ,  $y(t)$ ,  $z(t)$  and  $dy/dt$  on a graph for 15 days using a step size of 0.1 days. [Hint: to plot  $dy/dt$ , use your values of  $x$  and  $y$  to compute  $dydt = \lambda xy - \mu y$  and plot  $dydt$ .]
- (2) Suppose your discrete data has  $x(39) = 22$  and  $x(40) = 19$ , at what time  $t_{20}$  does  $x(t_{20}) = 20$ ? You need to use linear interpolation to find  $t_{20}$ . Theoretically you find the equation of the line through the data points  $(39, 22)$  and  $(40, 19)$  and find where this line intersects the horizontal line  $x = 20$ . Alternately you can use the formula  $t_{20} = [(22 - 20)40 + (20 - 19)39]/(22 - 19)$  which is 2/3 of the way from 39 to 40.
- (3) The scilab command 'max' returns the maximum value of discrete data and the first index it occurred on.

$$[m, i] = \max(y) \text{ when } y = [ 5 \ 7 \ 9 \ 8 \ 7 \ 9 \ 3 ]$$

will return  $m = 9$ ,  $i = 3$ .

- (4) Include the scilab code you needed to use. Include the commands used to generate the data. Do **NOT** include errors **NOR** long meaningless strings of output. Be sure to state the scilab version used and your compute type and OS.
- (5) Study the effect of the parameter  $\lambda$  by comparing the plot of  $y(t)$  above with the plots of other values of  $\lambda$ , and filling out a table like the one below. Note that your table entries should not be rounded off, they should have as many digits as scilab gives by default. Your plot should not plot the  $x$  or  $z$  value.

$\lambda$	$\mu/\lambda$	peak $y(t)$	peak time	$x$ at peak time	when $x(t) = 20$
0.001	?	?	?	?	?
0.002	?	?	?	?	?
0.003	?	?	?	?	?

- (6) Study the effect of the parameter  $\mu$  by comparing the plot of  $y(t)$  above with the plots of other values of  $\mu$ , and by filling out a table like the one below. Note that

your table entries should not be rounded off, they should have as many digits as scilab gives by default. Your plot should not plot the  $x$  or  $z$  values.

$\mu$	$\mu/\lambda$	peak $y(t)$	peak time	$x$ at peak time	when $x(t) = 20$
1/18	?	?	?	?	?
1/14	?	?	?	?	?
1/10	?	?	?	?	?

- (7) We expect  $y$  to have a maximum value when  $\frac{dy}{dt} = 0$  or looking at the system when  $\lambda x(t)y(t) - \mu y(t) = 0$ , that is when  $x(t) = \mu/\lambda$ . Explain why the  $x$  at the maximum values doesn't match the  $\mu/\lambda$  value better.
- (8) The crude measure given by  $\mu/\lambda$  estimate suggests that

$$\mu = 1/7, \lambda = 0.001 \text{ and } \mu = 1/14, \lambda = 0.0005$$

will be roughly equivalent. Plot these two  $y$  values on the same plot and compare them. (You might need to look at a longer time period than 15 days.) Do they seem equivalent? Which is better and why do you think it is better.

Your lab 9 assignment must be a single stapled hardcopy document, no electronic files will be accepted. Note that the assignment is due at 3pm Friday 31 Oct.