MAP 3306 eMath2 **Test 2** 26 Oct 2007

Directions: Show **ALL** work for credit; Give **EXACT** answers when possible; Start each problem on a **SEPARATE** page; Use only **ONE** side of each page; Be neat; Leave margins on the left and top for the **STAPLE**; Nothing written on this page will be graded;

1. Match the PDE (I) – (V) to its characteristics (A) – (H): (A, B are elliptic; C, D parabolic and E– H hyperbolic.)

Equation		Characteristics Φ and Ψ
$I.$	$u_{xx} + 2u_{xy} = 0$	A. $y + x \pm ix$
$II.$	$-u_{xx} + 2u_{xy} = 0$	B. $y - x \pm ix$
$III.$	$u_{xx} + 2u_{xy} + 2u_{yy} = 0$	C. $y + x \pm 0$
$IV.$	$u_{xx} - 2u_{xy} + u_{yy} = 0$	D. $y - x \pm 0$
$V.$	$-2u_{xy} + u_{yy} = 0$	E. $y + x \pm x$
		F. $y - x \pm x$
		G. $x + y \pm y$
		H. $x - y \pm y$

2. Single traveling wave: consider the initial value problem (IVP):

$$c^2 u_{xx} = u_{tt};\ u(x,0) = f(x);\ u_t(x,0) = g(x)$$

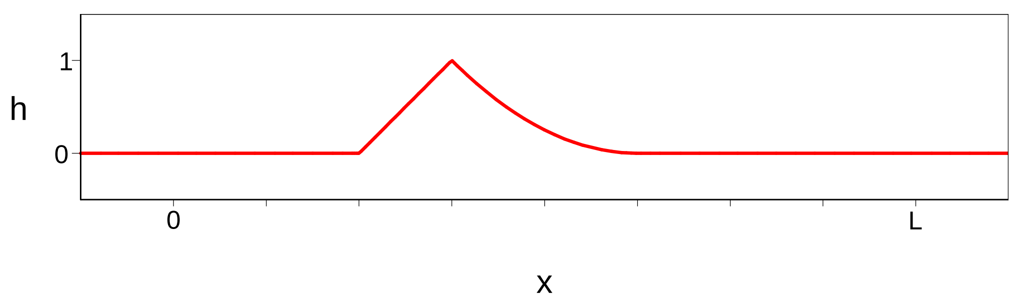
   (a) Show $w(x,t) = h(x - ct)$ is a solution to $c^2 u_{xx} = u_{tt}$.

   (b) Compute the initial values of the IVP, the $f(x)$ and $g(x)$, for the solution given in Part 2a.

   (c) Write D'Alembert's solution to the IVP.

   (d) Compute the integral term in Part 2c using functions in 2b.

   (e) Suppose $h(x)$ looks like the graph below and the boundary conditions at the endpoints $x = 0$ and $x = L$ are the usual fixed endpoints $u(0,t) = u(L,t) = 0$. Will the wave bounce back from the endpoint (which it must, because of conservation of energy) above or below the x-axis?



3. Find the solution to the one dimensional heat equation (with insulated ends) ($c^2 = L = 1$): namely

$$u_{xx} = u_t; \quad u_x(0,t) = u_x(1,t) = 0; \quad u(x,0) = 1 - 2x$$

There is more test on the other side.

4. True or False and a brief reason why or why not. The following trig identities might be useful

$$\sin^3 x = \frac{1}{4}(3\sin x - \sin 3x)$$
$$\cos^3 x = \frac{1}{4}(3\cos x + \cos 3x)$$

(a) The first step to solving the PDE $u_{xy} + u_y + x = 0$ like an ODE is to use $p = u_x$.

(b) To convert the PDE $u_x = 4yu$ into an ODE in $\frac{dw}{dt}$ is the transformation $u \to w;\ y \to t$.

(c) Any steady state solutions of the heat equation $c^2 u_{xx} = u_t$ has the form $Ax + B$

(d) A semi-infinite laterally insulated rod starts at the origin $x = 0$ and continues along the positive x-axis. The rod has been kept at a temperature of 100 for a long time. A time $t = 0$, the rod at $x = 0$ is suddenly changed to a temperature of 0 and kept at 0. A BVP modelling this might require some conditional at $x = \infty$, but it needs to include

$$c^2 u_{xx} = u_t;\ u(0,t) = 0;\ u(x,0) = 100$$

(e) The solutions of $u_x + u_y = 0$ of the form $X(x)Y(y)$ are $u(x,y) = \exp(k(x+y))$

(f) If $f(x) = \sum_{n=1}^{N} b_n \sin nx$, then the general solution to $100y'' + y = f(x)$ is

$$y(x) = A\cos\frac{x}{10} + B\sin\frac{x}{10} + \sum_{n=1}^{N} \frac{b_n}{1 - 100n^2}\sin nx$$

(g) If one of the two functions $f(x)$ and $g(x)$ is odd and the other is even and $a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} g(x)\,dx = 0$, then $g(x)$ is odd and $f(x)$ is even.

(h) If $\xi = 2x + 3y$ and $\eta = 3x - 2y$, then $u_{xy} = 6u_{\xi\xi} + 5u_{\xi\eta} - 6u_{\eta\eta}$

(i) The solution to the wave equation

$$u_{xx} = u_{tt};\ u(0,t) = u(\pi,t) = 0;\ u(x,0) = 4\sin^3 x;\ u_t(x,0) = 0$$

is $u(x,t) = 3\sin x\cos t - \sin 3x\cos 3t$

(j) The PDE $u_{xx} - u_x + u = \sin(xyu)$ is linear.