MAP 3306 eMath2 **Test 1** 28 Sep 2007

Directions: Show **ALL** work for credit; Give **EXACT** answers when possible; Start each problem on a **SEPARATE** page; Use only **ONE** side of each page; Be neat; Leave margins on the left and top for the **STAPLE**; Nothing written on this page will be graded;

1. Match the function (A) – (E) to its Fourier series (I) – (V):

A. $f(x) = \frac{\pi^2}{4}(1-2|x|) \quad (-1 < x < 1)$

B. $f(x) = \begin{cases} \frac{\pi}{4} & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ -\frac{\pi}{4} & \frac{\pi}{2} < x < \frac{3\pi}{2} \end{cases}$

C. $f(x) = \frac{x}{2} \quad (-\pi < x < \pi)$

D. $f(x) = \frac{\pi}{4}|x| \quad (-\pi < x < \pi)$

E. $f(x) = \begin{cases} \frac{\pi}{8}x(\pi - x) & 0 < x < \pi \\ \frac{\pi}{8}x(\pi + x) & -\pi < x < 0 \end{cases}$

I. $f(x) = \frac{\sin x}{1^3} + \frac{\sin 3x}{3^3} - \frac{\sin 5x}{5^3} + \cdots$

II. $f(x) = \frac{\sin x}{1} - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + \cdots$

III. $f(x) = \frac{\cos \pi x}{1^2} + \frac{\cos 3\pi x}{3^2} + \frac{\cos 5\pi x}{5^2} + \cdots$

IV. $f(x) = \frac{\cos x}{1} - \frac{\cos 3x}{3} + \frac{\cos 5x}{5} + \cdots$

V. $f(x) = \frac{\pi^2}{8} - \frac{\cos x}{1^2} - \frac{\cos 3x}{3^2} - \frac{\cos 5x}{5^5} + \cdots$

2. Find the general solution to the PDE $u_{xx} = 4y^2 u$ given that $u = u(x,y)$ is function of two variables.

3. Find the Fourier series of the function

$$f(x) = \begin{cases} \pi - 2x & 0 < x < \pi \\ \pi + 2x & -\pi < x < 0 \end{cases}$$

4. True or False and a brief reason why or why not. The following trig identities might be useful

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$
$$\cos^3 x = \frac{1}{4}(\cos 3x + 3\cos x)$$

(a) For integers n, $e^{-in\pi/2} = i^n$

(b) When $n \neq 0$, $\int e^{ix/n}\,dx = -ine^{ix/n} + C$

(c) The best trig poly approx with $N = 2$ for $f(x) = \cos^3 x$ is $\frac{3}{4}\cos x$

(d) Parseval's identity applied to $\cos^3 x$ will imply

$$\int_0^\pi \cos^6 x\,dx = \frac{5\pi}{16}$$

(e) The fundamental period of $\sin(n\pi x/L)$ is L/n

(f) If $f(x)$ has Fourier series $\sum n^{-2}\cos nx$, then $g(x) = f(x+\pi)$ has Fourier series $\sum(-1)^n n^{-2}\cos nx$

(g) Every function with Fourier series $\sum n^{-2}\cos nx$ is continuous.

(h) If $\lim_{x\to 0^-} f(x) = A < B = \lim_{x\to 0^+} f(x)$, then $f(x)$ cannot be piecewise smooth.

(i) $u(x,y) = \exp(xy)$ is a solution to $x^2u_{xx} - 2xyu_{xy} + y^2u_{yy} = 0$

(j) The second order PDE $x^2u_{xx} - 2xyu_{xy} + y^2u_{yy} = \sin(x^2)$ is linear.