

**Directions:** Show **ALL** work for credit; Give **EXACT** answers when possible; Start each problem on a **SEPARATE** page; Use only **ONE** side of each page; Be neat; Leave margins on the left and top for the **STAPLE**; Nothing written on this page will be graded;

1. Match each function (I–VI) to all the ODE's (A–F) it solves:

$$I. y = e^x \quad II. y = e^{-x} \quad III. y = \sin x \quad IV. y = \sinh x \quad V. y = xe^x \quad VI. y = e^{-2x}$$

$$y'' + y = 0 \quad A$$

$$y'' - y = 0 \quad B$$

$$y'' - y' = 0 \quad C$$

$$y'' - y' + 2y = 0 \quad D$$

$$y'' - 2y' + y = 0 \quad E$$

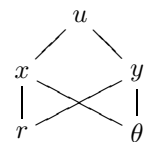
$$y' + y = 0 \quad F$$

2. The function  $u(x, y)$  can also be considered a function of the polar coordinates  $(r, \theta)$  via  $x = r \cos \theta$  and  $y = r \sin \theta$ . Using the chain rule once to find  $\frac{\partial u}{\partial r} = u_r$  gives

$$u_r = u_x x_r + u_y y_r = u_x \cos \theta + u_y \sin \theta$$

and  $u_r$  is expressed in terms of just  $u_x$  and  $u_y$ . Use the chain rule again to find the  $u_{r\theta}$  in terms of  $u_{xx}, u_{xy}, u_{yy}, u_x$  and/or  $u_y$ .

Hint



3. Find the general solution to  $y'' - y' + 6y = e^x$
4. True or False and a brief reason why or why not.
- $\int \cos x \, dx = -\sin x + C$  and  $\int \sin x \, dx = \cos x + C$
  - $\int_{-\pi}^{\pi} \sin(x + x^3 + x^5) - x \cos(\sin(x)) \, dx = 0$
  - The trigonometric functions  $\sin 2x$ ,  $\cos 2x$ , and  $\tan x$  all have fundamental period  $\pi$ .
  - The following are trig identities  $\sin(x + y) = \sin x \cos y + \cos x \sin y$  and  $\cos(x + y) = \cos x \cos y + \sin x \sin y$
  - $e^{i\theta} = \sin \theta + i \cos \theta$
  - The ODE  $y'' + y' - 3y = \sin y$  is linear.
  - The function  $\mu = x$  is the integrating factor needed for the ODE  $y' + y/x = \sin x$ .
  - The values  $r$  for which  $y = x^r$  is a solution of  $x^2 y'' - 2xy' + 2y = 0$  are  $r = 1$  and  $r = 2$ .
  - $u(x, t) = e^{-t} \sin 5x$  is a solution to the PDE  $u_{xx} = 25u_t$
  - If  $u(x, y) = f(2x - 3y)$  then  $u_x = 2f(2x - 3y)$