MAP 3306 eMath2 **Quiz 9** 21 Nov 2007 Name: ______________________________

Directions: Show **ALL** work for credit; Give **EXACT** answers when possible; **SIMPLIFY** your answers;

1. Use Fourier transforms to find the solution of $u(x,t)$ of the PDE $3u_x + u_t + 2u = 0 \qquad u(x,0) = f(x)$. The equations on the other side of the page might be helpful.

$$\mathscr{F}[f(x)] = \widehat{f}(w) \text{ or simply } \mathscr{F}[f] = \widehat{f} \tag{1}$$

$$\mathscr{F}^{-1}[\widehat{f}(w)] = f(x) \text{ or simply } \mathscr{F}^{-1}[\widehat{f}] = f \tag{2}$$

$$\mathscr{F}[f(x)](w) = \widehat{f}(w) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-iwx}\, dx \tag{3}$$

$$\mathscr{F}^{-1}[\widehat{f}(w)](x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \widehat{f}(w) e^{iwx}\, dw \tag{4}$$

$$\mathscr{F}[u(x,t)](w,t) = \widehat{u}(w,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} u(x,t) e^{-iwx}\, dx \tag{5}$$

$$\mathscr{F}^{-1}[\widehat{u}(w,t)](x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \widehat{u}(w,t) e^{iwx}\, dw \tag{6}$$

$$\mathscr{F}[af(x) + bg(x)](w) = a\widehat{f}(w) + b\widehat{g}(w) \tag{7}$$

$$\mathscr{F}[f'(x)](w) = iw\widehat{f}(w) \tag{8}$$

$$\mathscr{F}[f''(x)](w) = -w^2\widehat{f}(w) \tag{9}$$

$$\mathscr{F}[\frac{\partial}{\partial x} u(x,t)](w,t) = iw\widehat{u}(w,t) \tag{10}$$

$$\mathscr{F}[\frac{\partial^2}{\partial x^2} u(x,t)](w,t) = -w^2\widehat{u}(w,t) \tag{11}$$

$$\mathscr{F}[\frac{\partial}{\partial t} u(x,t)](w,t) = \frac{\partial}{\partial t}\widehat{u}(w,t) \tag{12}$$

$$\mathscr{F}[\frac{\partial^2}{\partial t^2} u(x,t)](w,t) = \frac{\partial^2}{\partial t^2}\widehat{u}(w,t) \tag{13}$$

$$[f * g](x) = \int_{-\infty}^{\infty} f(w) g(x-w)\, dw = [g * f](x) = \int_{-\infty}^{\infty} f(x-w) g(w)\, dw \tag{14}$$

$$\mathscr{F}[f * g] = \sqrt{2\pi}\widehat{f}\widehat{g} \tag{15}$$

$$f(x-a) = \mathscr{F}^{-1}[e^{-iwa}\widehat{f}(w)] \tag{16}$$

$$\mathscr{F}[\exp(-ax^2)] = \frac{1}{\sqrt{2a}} \exp(\frac{-w^2}{4a}) \tag{17}$$

$$\sin wa = \frac{e^{iwa} - e^{-iwa}}{2i} \tag{18}$$

$$\cos wa = \frac{e^{iwa} + e^{-iwa}}{2} \tag{19}$$

$$\frac{1}{\sqrt{2\pi}} \int_{-a}^{a} e^{-iwx}\, dx = \sqrt{\frac{2}{\pi}} \frac{\sin aw}{w} \tag{20}$$

$$\mathscr{F}\left[\frac{\sin ax}{x}\right] = \sqrt{\frac{\pi}{2}} \text{ if } |w| < a; 0 \text{ otherwise} \tag{21}$$

$$\frac{1}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-ax} e^{-iwx}\, dx = \frac{1}{\sqrt{2\pi}(a+iw)} \tag{22}$$