MAP 2302 Diff-E-Qs **Test 4** 29 Nov 2006

Directions: Show **ALL** work for credit; Give **EXACT** answers when possible; Start each problem on a **SEPARATE** page; Use only **ONE** side of each page; Be neat; Leave margins on the left and top for the **STAPLE**; Calculators can be used for graphing and calculating only; Nothing written on this page will be graded;

1. Solve using Laplace transforms [see other side for transform table]

$$y'' + 4y = \delta(t - \pi) \quad y(0) = 1;\ y'(0) = 2;$$

2. Find the following transforms or inverse transforms:

   (a) $\mathcal{L}\{t^3 e^{7t}\}$

   (b) $\mathcal{L}\{f'''(t)\}$ in terms of $F(s) = \mathcal{L}(f(t))$ and initial values of f.

   (c)
$$\mathcal{L}^{-1}\left\{\frac{s+1}{s^2-4}\right\}$$

   (d)
$$\mathcal{L}^{-1}\left\{\frac{s+2}{(s-3)^2+4}\right\}$$

   (e) $\mathcal{L}\{u_5(t)(t+2)\}$

3. True or False and a brief reason why or why not.

   (a) The ODE $(1-t^2)y'' + t/y' + t^3 y = 0$ is linear.

   (b) The radius of convergence of $\sum_{n=0}^{\infty}(-2)^n x^n/(n+3)$ is $1/2$.

   (c) $\frac{1}{1+x^2} = \sum_{n=0}^{\infty} x^{2n}$.

   (d) $\exp(4x) = \sum_{n=0}^{\infty} 4^n x^n/n!$.

   (e) For all t, $(u_1(t) - u_3(t))(u_2(t) - u_4(t)) = u_2(t) - u_3(t)$

   (f) The function $y(t) = u_5(t)\tan(t-5)$ is continuous at $t = 5$.

   (g) The series solution about $x_0 = 1$ for $y'' + \frac{3}{x^2+1}y' + \frac{5}{(x-2)(x+3)}y = 0$ has a radius of converence $\rho \geq \sqrt{2}$

   (h) If $y(x) = \sum_{n=0}^{\infty} a_n x^n$, then $y'''(0) = a_3$.

   (i) The recurence relation for IVP with $y(0) = 0;\ y'(0) = 1$ is $(n+2)(n+1)a_{n+2} - (n+1)a_n = 0$ for $n \geq 0$, the series solution is

$$y(x) = \frac{x}{1} + \frac{x^3}{1\cdot 3} + \frac{x^5}{1\cdot 3\cdot 5} + \cdots + \frac{x^{2n+1}}{1\cdot 3\cdots(2n+1)} + \cdots$$

   .

   (j) For all t, $u_1(t)(t-1) - u_2(t)(t-2) = \int_0^t u_1(x) - u_2(x)\,dx$

4. Find the first four non-zero terms of the series solution about $x_0 = 0$ to the IVP

$$(1-x^2)y'' - xy' + y = 0,\ y(0) = 3,\ y'(0) = 0$$