MAP 2302 Diff-E-Qs **Test 2** 11 Oct 2006

Directions: Show **ALL** work for credit; Give **EXACT** answers when possible; Start each problem on a **SEPARATE** page; Use only **ONE** side of each page; Be neat; Leave margins on the left and top for the **STAPLE**; Calculators can be used for graphing and calculating only; Nothing written on this page will be graded;

1. For the second order ODE $y'' - 25y = 0$
   (a) Find the solution y_1 with inital values $y_1(0) = 1, y_1'(0) = 0$
   (b) Find the solution y_2 with inital values $y_2(0) = 0, y_2'(0) = 1$

2. For the homogenuous equations A–E complete a table like the one below. In the first column is the letter A–E, in the second column write the roots to the characteristic polynomial, in the third column write the general solution to the homogenuous problem and in the fourth column decide if all solutions eventually damp to zero.

Equation Letter	Roots Char Poly	Gen Solution $y(t)$	Is $\lim_{t\to\infty} y(t) = 0$ always
A,B,C,D or E	?	?	?

$$\begin{aligned} y'' - 6y' + 8y &= 0 \quad (A)\\ y'' + 2y' + 2y &= 0 \quad (B)\\ y'' + 4y' + 4y &= 0 \quad (C)\\ y'' + y' - 12y &= 0 \quad (D)\\ y'' &= 0 \quad (E) \end{aligned}$$

3. True or False and a brief reason why or why not.
   (a) The ODE $y'' + ty'y = \sin t$ is linear.
   (b) $e^{5\pi i} = -1$
   (c) If $y_1(t)$ and $y_2(t)$ are both solutions to the linear ODE $L[y] = 0$ then the general solution can be written as $C_1y_1(t) + C_2y_2(t)$.
   (d) The Wronskian $W(e^t, e^{-t})$ is constant.
   (e) If the Wronskian of $f(t)$ and $g(t)$ is non-zero at one point t_0, then $f(t)$ and $g(t)$ are linearly independent.
   (f) If the constant $b > 0$, then any solution $y(t)$ to the ODE $y'' + by' + y = 0$ satisfies
   $$\lim_{t\to\infty} y(t) = 0$$
   (g) The solution to the IVP $y'' + y = 0$; $y(0) = A, y'(0) = B$ is $y(t) = A\sin(t) + B\cos(t)$.
   (h) $y(t) = (e^{-1})te^t$ is a solution to the inital value problem $y'' - 2y' + y = 0$; $y(1) = 1, y'(1) = 2$
   (i) If $L[y] = y'' + y' + 2y$ then $L[e^{-2t}] = 8e^{-2t}$.
   (j) The functions $5 + 5\tan^2 t$ and $1/\cos^2 t$ are linearly independent.

4. Exact and Euler
   (a) Show this IVP is exact $x + y + xy' = 0$; $y(2) = 5$ and solve it using the exact method explicitly for y.
   (b) Use Euler's method on $y' = y - t$ with $\Delta t = 2$ and $y(2) = 5$ to fill in a table like the one below.

t	$y(t)$	$y'(t)$	Δy	$y(t + \Delta t)$
2	?	?	?	?
4	?	?	?	?
6	?	?	?	?