MAP 2302 Diff-E-Qs **Test 1** 20 Sep 2005

Directions: Show **ALL** work for credit; Give **EXACT** answers when possible; Start each problem on a **SEPARATE** page; Use only **ONE** side of each page; Be neat; Leave margins on the left and top for the **STAPLE**; Calculators can be used for graphing and calculating only; Nothing written on this page will be graded;

1. For the function $f(y) = (y-1)(y-5)$ and the autonomous ODE $\frac{dy}{dt} = f(y)$, determine the critical (equilibrium) points, and classify each one as stable, unstable or semistable. Draw the phase line and sketch several graphs of solutions in the ty-plane. (At least one in each "region" of the phase line, and be sure to get the concavity and the inflection points correct. Hint: graph $f(y)$)

2. Solve the IVP problems: (explicitly solve for y)

$$\text{(A) } y' = \frac{2x}{y + x^2 y} \qquad y(0) = -2 \qquad\qquad \text{(B) } ty' + 2y = 4t^2 \qquad y(1) = 2$$

3. True or False and a brief reason why or why not.
   (a) The ODE $y'' + \sqrt{x}y' - 3x^2 e^x y = 3x^x$ is linear.
   (b) The ODE $(y')^2 + y''' + y^2 = \sin t \cos t$ is second order.
   (c) The function $t^6 + C/t^6$ is the general solution of $ty' - 6y = 0$.
   (d) If the rate of lemonade flowing into a tank is 5 liters per minute and the rate of lemonade flowing out of the tank is 6 liters per minute, then the volume of lemonade in the tank satisfies
   $$\frac{dV}{dt} = -1$$
   (e) The function e^{2t} is a solution to $y' - e^{-4t}y^3 = e^{2t}$.
   (f) Both $y = 0$ and $y = (\frac{3}{2}t)^{2/3}$ are solutions to the intial value problem $y' = y^{1/3}; y(0) = 0$.
   (g) $y = \tan t$ is a solution to the IVP $y' = 1 + y^2; y(\pi/4) = 1$.
   (h) There are polynomials $p(t)$ and $q(t)$ so that $y = \tan t$ is a solution to $y' + p(t)y = q(t)$.
   (i) The equations $x' = -xy; y' = 1 - x'; x(0) = y(0) = 0$ are a PDE.
   (j) If $y = 0$ is a stable equilibrium solution of the autonomous $y' = f(y)$, $f(y)$ and $f'(y)$ are continuous everywhere, and $y(t)$ is the unique solution of the IVP $y' = f(y); y(0) = 1$, then $\lim_{t\to\infty} y(t) = 0$.

4. Consider a pond A with a constant volume V of water. The pond contains an amount $Q(t)$ pounds of salt, which is always evenly distributed throughout the pond. Assume fresh water enters the pond at the rate of 100 gallons per second and that water leaves the pond at the same rate. Initially there are 100 pounds of salt in the pond. Your answers to the questions below might have the parameter V standing for the constant volume of the water in the pond in gallons.
   (a) Write an IVP for $Q(t)$.
   (b) Solve your IVP.
   (c) Compute how long it will take for the amount of salt in the pond to drop to $100/e$ pounds (≈ 37) pounds?
   (d) Pond A drains into a pond B of the same constant volume V of water. (So water enters and leaves pond B also at the rate of 100 gallons per second.) Pond B initially contains only fresh water. Write (but do **NOT** solve) an IVP for $S(t)$ the amount of salt in pond B