

**Directions:** Use only **ONE** side of each page, use ink and a staple. This lab should resonate resonance with good vibrations.

This lab is about linear approximations to non-linear second order ODEs. On page 21 of the text, the equation of an undamped pendulum is derived. We consider the unforced damped equation

$$y'' + y' + \sin y = 0 \quad (A)$$

where  $y$  is the angle  $\theta$  the pendulum makes with the horizontal. Since the Taylor series of

$$\sin y \approx y - \frac{y^3}{6} + \frac{y^5}{120} + \dots$$

we have a number of approximations. The most important is the linear approximation

$$y'' + y' + y = 0 \quad (B)$$

We also consider the fifth order approximation

$$y'' + y' + y - \frac{y^3}{6} + \frac{y^5}{120} = 0 \quad (C)$$

and briefly the third order approximation

$$y'' + y' + y - \frac{y^3}{6} = 0 \quad (D)$$

to expose short comings that truncations can cause.

We consider the initial value problem  $y(0) = 0$  and  $y'(0) = v$  where  $v$  takes on the increasing sequence of values 1, 3 and 5.

Don't forget to explain how you got your numbers from scilab Also remember clarity and presentation.

1. Consider the case  $v = 1$ , In this range the equations  $A$ ,  $B$  and  $C$  closely agree. Complete the table below to show this closeness. The next equilibrium is the next time  $t$  when  $y(t) = 0$ , likely you will need to interpolate to find  $t$ .

equation	maximum (t,y(t))	next equilibrium	minimum (t,y(t))	next equilibrium
$A$	(?, ?)	$t = ?$	(?, ?)	$t = ?$
$B$	(?, ?)	$t = ?$	(?, ?)	$t = ?$
$C$	(?, ?)	$t = ?$	(?, ?)	$t = ?$

2. Next consider  $v = 3$ , in this range  $A$  and  $C$  agree well but differ somewhat from  $B$ . The next equilibrium is the next time  $t$  when  $y(t) = 0$ . Complete the same table

equation	maximum (t,y(t))	next equilibrium	minimum (t,y(t))	next equilibrium
$A$	(?, ?)	$t = ?$	(?, ?)	$t = ?$
$B$	(?, ?)	$t = ?$	(?, ?)	$t = ?$
$C$	(?, ?)	$t = ?$	(?, ?)	$t = ?$

3. All bets are off by the time  $v = 5$ . Explain what happens for  $A$  in particular the limiting value as  $t \rightarrow \infty$ . Compare to  $B$ .
4. All bets are off by the time  $v = 5$ . Explain what happens for  $D$  in particular the limiting value as  $t \rightarrow \infty$ . Compare to  $C$ .