MAP 2302 Diff-E-Qs **Euler/RK Problems** Fall 2006

Directions: Use only **ONE** side of each page; Be neat; Leave margins on the left and top for the **STAPLE**; Nothing written on this page will be graded;

1. For the IVP $y' = t^2 + y^2 = f(t, y)$ and $y(0) = 1$ numerically (by hand) compute the solution with a stepsize of $\Delta t = h = 1/2$.

   (a) By Euler's method by completing a table like the one below. See the example in part 2. Euler's method uses $hy'(t) = hf(t, y(t)) \approx \Delta y$ and so $y(t + h) \approx y + \delta y = y(t) + hy'(t)$.

t	$y(t)$	$y'(t)$	Δy	$y(t+h)$
0				
1/2				
1				
3/2				

   (b) By RK's method by completing a table like the one below. RK also uses $y(t + h) \approx y + \Delta y$ but $\Delta y = (\text{avg } y')h$ where

$$k_1 = f(t, y(t)) \quad k_2 = f(t{+}h/2, y(t){+}k_1h/2) \quad k_3 = f(t{+}h/2, y(t){+}k_2h/2) \quad k_4 = (f(t{+}h), y(t){+}k_3h)$$

$$\text{avg } y' = (k_1 + 2k_2 + 2k_3 + k_4)/6$$

t	$y(t)$	k_1	k_2	k_3	k_4	avg y'	Δy	$y(t+h)$
0								
1/2								

2. We do an example that is almost identical. Our IVP is $y' = 1 + y^2 = f(t, y)$ and $y(0) = 0$. Observe, first $y(t) = \tan t$ is a solution to this equation and second that this solution blows up as $t \to \pi/2$ from the left.

   (a) Euler's method table. Euler's method uses $hy'(t) = hf(t, y(t)) \approx \Delta y$ and so $y(t{+}h) \approx y(t){+}hy'(t)$.

t	$y(t)$	$y'(t)$	Δy	$y(t+h)$
0	0	1	1/2	1/2
1/2	1/2	5/4	5/8	9/8
1	9/8	145/64	145/128	289/128
3/2	289/128	99905/16384	99905/32768	173889/32768

   (b) RK's method table. RK uses

$$k_1 = f(t, y(t)) \quad k_2 = f(t{+}h/2, y(t){+}k_1h/2) \quad k_3 = f(t{+}h/2, y(t){+}k_2h/2) \quad k_4 = f(t{+}h, y(t){+}k_3h)$$

$$\text{avg } y' = (k_1 + 2k_2 + 2k_3 + k_4)/6$$

t	$y(t)$	k_1	k_2	k_3	k_4	avg y'	Δy	$y(t+h)$
0	0	1	1.062500000	1.070556641	1.218132667	1.080707658	0.5403538290	0.5403538290
0.5	b	1.291982260	1.745372176	1.953936782	1.634010036	1.720768369	0.8603841845	1.400738014

b is used here for 0.5403538290 so the table will fit on the page.