MAC 2313 Calculus 3

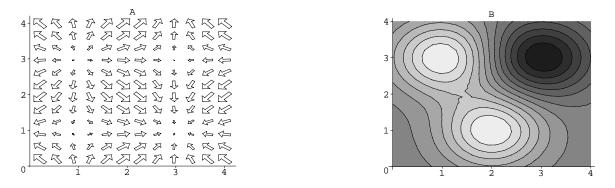
Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only. Give exact answers when possible.

1. Use the Chain Rule to find $\partial z/\partial u$ and $\partial z/\partial v$ when $z = \sin(x/y)$, $x = \ln u$ and $y = u^2 - v^2$.

2. Fixing Maple errors. Each of the following produced an error or an empty graph, explain how to fix each.

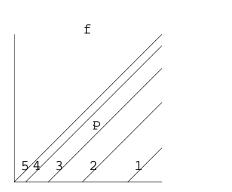
- a. $plot3d(exp^x*sin(y),x=0..1,y=0..2*Pi);$
- b. $plot3d(x^2-x^*y+y^2,x=0..1,y=1..1);$
- c. $f=\sin(x)^*y^2+x^2*\sin(y)$;plot3d(f,x=-1..1,y=-1..1);
- d. plot3d(x y,x=-1..1,y=-1..1);
- e. plot3d(sin(x)*sin(y),x=0..pi,y=0..pi);
- 3. For the function $f(x, y) = x^3 + xy + y^2$
 - a. Compute the quadratic Taylor polynomial for f at the point (-1, 2).
 - b. Compute the equation of the normal line to f at the point (-1, 2).

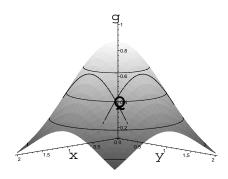
4. The graph A is a plot of ∇f , the gradient of f and the graph B is a contourplot of g. (Light regions have higher values than dark regions.] Find the co-ordinates of all extrema of f and g and **LABEL** them as either local minimums, local maximums or saddle points.



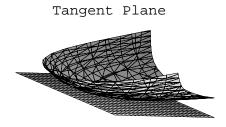
5. Find the directional derivative of $f(x, y, z) = 3x^2y^2 + 2yz$ as you leave the point (1, -1, 0) heading in the direction of the point (0, 1, 1).

6. The point P is on the contour graph of the function f (below left) and the point Q is on the surface of the graph of the function g (below right). Let **u** be the unit vector $\mathbf{u} = (-\mathbf{i} - \mathbf{j})/\sqrt{2}$. Find the sign (positive, negative or zero) of $f_x(P)$, $f_y(P)$, $f_{xx}(P)$, $f_{yy}(P)$, $f_{xy}(Q)$, $g_y(Q)$, $g_{xx}(Q)$, $f_u(P)$ and $g_u(Q)$.





There is more test on the back.



7. Check that the point (-1, 1, 2) lies on the surface $\cos(x + y) = e^{xz+2}$ and find the equation of the tangent plane to this surface at (x, y, z) = (-1, 1, 2)

8. Sketch the region of integration, reverse the order of integration and evaluate

$$\int_0^1 \int_{e^x}^e \frac{y}{\ln y} \, dy \, dx$$

9. Find critical points of the function $f(x, y) = (x + y)(x^2 + y^2 - 2)$. Classify these local extrema by filling out a table like the one below, with a separate line for each critical point. [Hint: Use your TI-89 to check that you got the correct collection of critical points.]

(x,y)	f_{xx}	f_{yy}	f_{xy}	big D	Classification
?	?	?	?	?	?

10. Use your TI-89 to plot the z = 1 contour of the function $z = g(x, y) = x^2 + xy + y^2$. On the same graph, plot some contour lines for f(x, y) = x + y. Use Lagrange Multipliers to find the maximum and minimum **VALUES** for f(x, y) on the constraint g(x, y) = 1.