MAC 2313 Calculus 3 **Test 2** 17 Oct 2002

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only. Give exact answers when possible.

1. Use the Chain Rule to find $\partial z/\partial u$ and $\partial z/\partial v$ when $z = \sin(x/y), x = \ln u$ and $y = u^2 - v^2$.

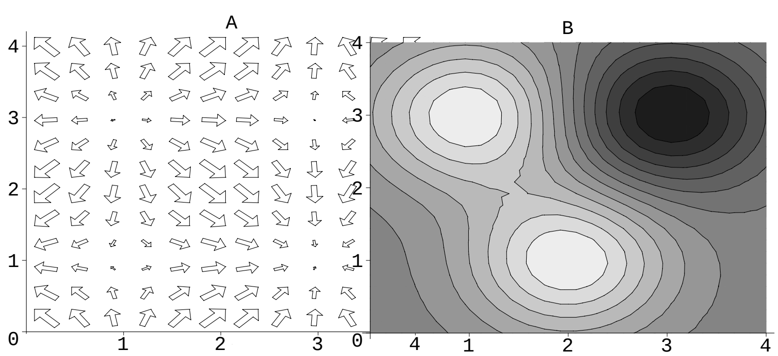
2. Fixing Maple errors. Each of the following produced an error or an empty graph, explain how to fix each.

- a plot3d(exp^x*sin(y),x=0..1,y=0..2*Pi);
- b plot3d(x^2-x*y+y^2,x=0..1,y=1..1);
- c f=sin(x)*y^2+x^2*sin(y);plot3d(f,x=-1..1,y=-1..1);
- d plot3d(x y,x=-1..1,y=-1..1);
- e plot3d(sin(x)*sin(y),x=0..pi,y=0..pi);

3. For the function $f(x,y) = x^3 + xy + y^2$

- a Compute the quadratic Taylor polynomial for f at the point $(-1, 2)$.
- b Compute the equation of the normal line to f at the point $(-1, 2)$.

4. The graph A is a plot of ∇f, the gradient of f and the graph B is a contourplot of g. (Light regions have higher values than dark regions.] Find the co-ordinates of all extrema of f and g and **LABEL** them as either local minimums, local maximums or saddle points.



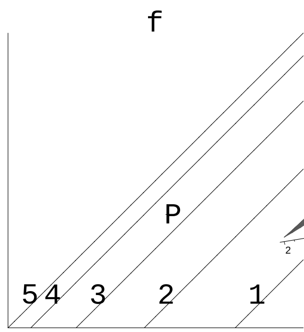
5. Find the directional derivative of $f(x,y,z) = 3x^2y^2 + 2yz$ as you leave the point $(1, -1, 0)$ heading in the direction of the point $(0, 1, 1)$.

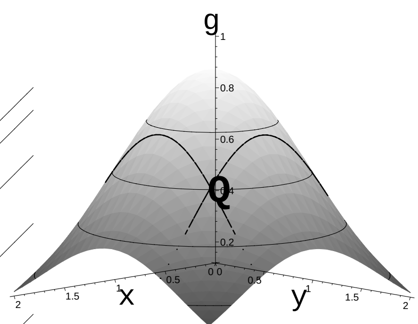
6. The point P is on the contour graph of the function f (below left) and the point Q is on the surface of the graph of the function g (below right). Let $\mathbf{u}$ be the unit vector $\mathbf{u} = (-\mathbf{i} - \mathbf{j})/\sqrt{2}$.

Find the sign (positive, negative or zero) of the partials and directional derivations:

$f_x(P), f_y(P), f_{xx}(P), f_{yy}(P), f_{xy}(P)$ $\qquad\qquad$ $g_x(Q), g_y(Q), g_{xx}(Q)$

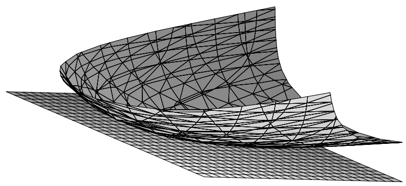
and $f_{\mathbf{u}}(P)$ and $g_{\mathbf{u}}(Q)$.





There is more test on the back.

Tangent Plane



7. Check that the point $(-1,1,2)$ lies on the surface $\cos(x+y) = e^{xz+2}$ and find the equation of the tangent plane to this surface at $(x,y,z) = (-1,1,2)$

8. Sketch the region of integration, reverse the order of integration and evaluate

$$\int_0^1 \int_{e^x}^{e} \frac{y}{\ln y}\, dy\, dx$$

9. Find critical points of the function $f(x,y) = (x+y)(x^2+y^2-2)$. Classify these local extrema by filling out a table like the one below, with a separate line for each critical point. [Hint: Use your TI-89 to check that you got the correct collection of critical points.]

(x,y)	f_{xx}	f_{yy}	f_{xy}	big D	Classification
?	?	?	?	?	?

10. Use your TI-89 to plot the $z=1$ contour of the function $z = g(x,y) = x^2+xy+y^2$. On the same graph, plot some contour lines for $f(x,y) = x+y$. Use Lagrange Multipliers to find the maximum and minimum **VALUES** for $f(x,y)$ on the constraint $g(x,y) = 1$.