13 Partial Information and the Puzzle of War

In Lecture 12 we took for granted, albeit implicitly, that the values of the parameters κ, b_1 and b_2 were common knowledge, so that either nation had full information about the other and hence would be able to compute the Nash equilibrium. Often, however, one of two nations will have only partial information about the other, and we consider that possibility in this lecture. We begin, however, by continuing to assume full information, in order to mathematize what has often been called the central puzzle of war—e.g., by Field and Briffa (2013, p. 321), who state it as the question, "why, when war is so self-evidently risky, costly and destructive, has it occurred with such regularity throughout history?" The mathemization is due to Fearon (1995).

Let us suppose that, as in Lecture 9, all possible status quos—and hence all possible outcomes of war—between two nations can be idealized as points of the unit interval $[0, 1]$, and that the utility for Nation 1 or Nation 2 of status quo x can be idealized as $U(x)$ or $V(x)$, respectively, where

$$U'(x) \;>\; 0, \qquad V'(x) \;<\; 0 \tag{13.1}$$

for all $x \in (0, 1)$ with

$$U(0) \;=\; 0 \;=\; V(1), \qquad U(1) \;=\; 1 \;=\; V(0). \tag{13.2}$$

Thus, as in Lecture 9, the left-hand extreme of $[0, 1]$ represents the best possible outcome for Nation 2 and worst for Nation 1, while the right-hand extreme represents the best possible outcome for Nation 1 and worst for Nation 2. Let us now follow Fearon (1995, p. 387) in further supposing that if these nations go to war, then Nation i incurs costs totalling $c_i(> 0)$ and wins with probability p_i, where

$$p_1 + p_2 \;=\; 1 \tag{13.3}$$

and c_i includes costs of any kind, measured as a loss of utility, so that

$$c_i \;\in\; (0, 1) \tag{13.4}$$

for both i. If Nation 1 wins then the outcome is 1, whereas if Nation 2 wins then the outcome is 0. So the expected utility of war is

$$p_1 U(1) + p_2 U(0) - c_1 \;=\; p_1 - c_1 \tag{13.5}$$

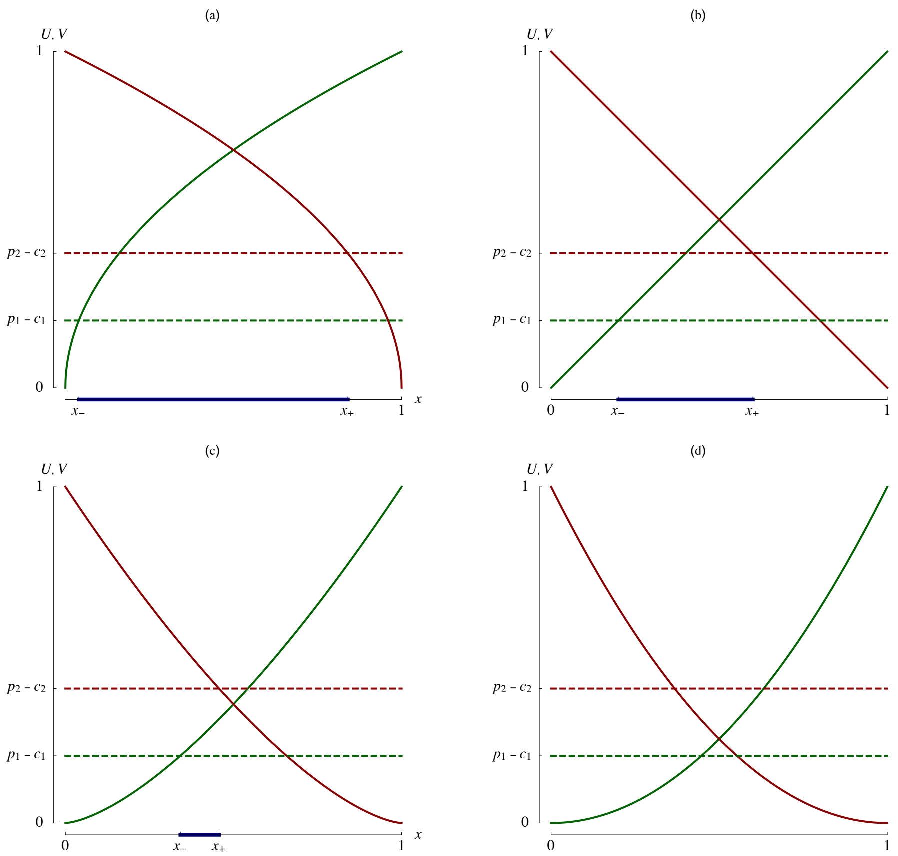
for Nation 1 and

$$p_1 V(1) + p_2 V(0) - c_2 \;=\; p_2 - c_2 \tag{13.6}$$

for Nation 2. If, instead of going to war, these two nations bargain their way to a new status quo x, then their utilities are $U(x)$ and $V(x)$, respectively. So both nations should prefer a new status quo x—and peace—to the outcome of war if both

$$U(x) \;\geq\; p_1 - c_1 \tag{13.7}$$

Figure 13.1: Negotiated outcomes that both sides prefer to fighting. The bargaining range B is indicated in blue for (a) $\rho_1 = \rho_2 = \frac{1}{2}$, (b) $\rho_1 = \rho_2 = 1$ and (c) $\rho_1 = \rho_2 = \frac{3}{2}$ in (13.9) with $p_1 = \frac{2}{5}$, $p_2 = \frac{3}{5}$ and $c_1 = c_2 = \frac{1}{5}$ in (13.7)–(13.8), whose left- and right-hand sides are shown solid and dashed, respectively, in green for U and in red for V; whereas $B = \varnothing$ for (d) $\rho_1 = \rho_2 = 2$ (and the same values of p_i, c_i as elsewhere).



and

$$V(x) \;\geq\; p_2 - c_2. \tag{13.8}$$

Let us satisfy (13.1)–(13.2) by setting

$$U(x) \;=\; x^{\rho_1}, \qquad V(x) \;=\; (1-x)^{\rho_2} \tag{13.9}$$

as in Lecture 9, so that ρ_i measures risk-proneness for Nation i (which is risk-averse, risk-neutral or risk-prone according to whether $\rho_i < 1$, $\rho_i = 1$ or $\rho_i > 1$, respectively). Then Figure 13.1 shows that there typically exists a status-quo interval

$$B \;=\; [x_-, x_+], \tag{13.10}$$

called the bargaining range, such that both sides prefer any $x \in B$ to war, where

$$x_- \;=\; U^{-1}(p_1 - c_1) \tag{13.11}$$

and

$$x_+ \;=\; V^{-1}(p_2 - c_2) \tag{13.12}$$

(and $^{-1}$ denotes an inverse, which must exist by 13.1). Indeed it is clear on geometrical grounds[1] that $B \neq \varnothing$ as long as neither nation is risk-prone; for example, if both nations are risk-neutral (Figure13.1(b)), so that $\rho_i = 1$ for both i, then (13.9)–(13.12) imply

$$\begin{aligned} B &= [p - c_1, 1 - p_2 + c_2] \\ &= [p_1 - c_1, p_1 + c_2] \end{aligned} \tag{13.13}$$

by (13.3). But $B \neq \varnothing$ may hold even if both nations are mildly risk-prone (Figure13.1(c)), although $x_- > x_+ \implies B = \varnothing$ for highly risk-prone nations (Figure13.1(d)). If typically there exists a bargaining range, then why is war ever considered rational, especially by nations that are either risk-neutral or risk-averse? This is in essence the puzzle of war.

A possible answer is partial information. For the sake of definiteness, let us suppose that Nation 1 changes the status quo in its favor—that is, increases the current value of x—by annexing some territory from Nation 2. Will Nation 2 accept this new status quo, or react by declaring war on Nation 1? From (13.8) and (13.12), if $x \leq x_+$, then Nation 2 should accept; whereas if $x > x_+$, then Nation 2 should declare war. Moreover, since larger x is better for Nation 1, it should annex just enough territory to increase the status quo all the way up to $x = x_+$. However, Nation 1 will know x_+ only if it knows both p_2 and c_2—and perhaps it doesn't.

For the sake of simplicity, let us first suppose that both nations are risk-neutral and that p_1, p_2 are common knowledge,[2] but that only Nation 2 knows c_2. Then, in view of (13.4), from Nation 1's perspective, c_2 is the realized value of a random variable, say Y, with a continuous distribution on $[0, 1]$. Let g_Y and G_Y denote its pdf and cdf, respectively. Then because Nation 2 will desist from war when (13.8) holds and otherwise declare war, and because $V(x) \geq p_2 - c_2$ for Nation 2 (which knows c_2) translates to $V(x) \geq p_2 - Y$ for Nation 1 (which doesn't know c_2), the payoff to Nation 1 from unilaterally shifting the status quo to x is the random variable

$$F_1 \;=\; \begin{cases} p_1 - c_1 & \text{if } V(x) < p_2 - Y \\ U(x) & \text{if } V(x) \geq p_2 - Y \end{cases} \;=\; \begin{cases} p_1 - c_1 & \text{if } Y < x - p_1 \\ x & \text{if } Y \geq x - p_1 \end{cases} \tag{13.14}$$

by risk-neutrality and (13.3), and so Nation 1's reward from choosing x is

$$\begin{aligned} f_1(x) &= E[F_1] \;=\; (p_1 - c_1) \cdot \text{Prob}(Y < x - p_1) + x \cdot \text{Prob}(Y \geq x - p_1) \\ &= \{p_1 - c_1\} G_Y(x - p_1) + x\,\{1 - G_Y(x - p_1)\} \\ &= x + \{p_1 - c_1 - x\} G_Y(x - p_1). \end{aligned} \tag{13.15}$$

Again for the sake of simplicity, let us now further assume that Y is uniformly distributed over $[0, 1]$, so that $g_Y(y) = 1$ or

$$G_Y(y) \;=\; y \tag{13.16}$$

[1] And formally proven by Fearon (1995, p. 410).

[2] In effect we have already made this assumption, since (13.3) could be false only if p_i were Nation i's subjective assessment of its probability of victory—as opposed to its objective probability of victory. For a discussion of this point, see, e.g., Mesterton-Gibbons (2007, pp. 291–293).

for $y \in [0,1]$. Then (13.15) reduces to

$$f_1(x) \;=\; x + (p_1 - c_1 - x)(x - p_1) \tag{13.17}$$

which is maximized by

$$x^* \;=\; \begin{cases} p_1 + \frac{1}{2}(1 - c_1) & \text{if } p_1 < \frac{1}{2}(1 + c_1) \\ 1 & \text{if } p_1 \geq \frac{1}{2}(1 + c_1) \end{cases} \tag{13.18}$$

with

$$f_1(x^*) \;=\; \begin{cases} p_1 + \frac{1}{4}(1 - c_1)^2 & \text{if } p_1 < \frac{1}{2}(1 + c_1) \\ 1 - (1 - p_1)(1 - p_1 + c_1) & \text{if } p_1 \geq \frac{1}{2}(1 + c_1). \end{cases} \tag{13.19}$$

The corresponding probability of war is

$$\begin{aligned} p_w \;&=\; \text{Prob}(V(x^*) < p_2 - Y) \;=\; \text{Prob}(Y < x^* - p_1) \\ &=\; G_Y(x^* - p_1) \;=\; x^* - p_1 \\ &=\; \begin{cases} \frac{1}{2}(1 - c_1) & \text{if } p_1 < \frac{1}{2}(1 + c_1) \\ 1 - p_1 & \text{if } p_1 \geq \frac{1}{2}(1 + c_1) \end{cases} \end{aligned} \tag{13.20}$$

which is always positive (unless Nation 1 has zero chance of winning a war), though it is small if c_1 is large. Thus partial information about the other nation's costs will favor war, even though there exists a negotiated solution that both sides would prefer.

Perhaps, however, only Nation 1 knows c_1, and only Nations 1 and 2 know p_1 and p_2. How would an external observer assess the overall probability of war between the two nations? One way to answer this question is to assume that (p_1, c_1) is uniformly distributed over the unit square—since we have no basis for assuming any other distribution. Then the overall probability of war can be assessed as

$$\begin{aligned} \text{E}[p_w] \;&=\; \int_0^1 \int_0^{\frac{1}{2}(1+c_1)} \tfrac{1}{2}(1 - c_1) \cdot 1 \, dp_1 \, dc_1 + \int_0^1 \int_{\frac{1}{2}(1+c_1)}^1 (1 - p_1) \cdot 1 \, dp_1 \, dc_1 \\ &=\; \tfrac{5}{24} \;\approx\; 0.2083, \end{aligned} \tag{13.21}$$

where E denotes expected value.

Let us now relax the assumption that p_1, p_2 are common knowledge between Nations 1 and 2, and assume instead that only Nation i knows p_i, which therefore becomes a subjective assessment of its probability of victory—as opposed to an objective one. So we can no longer assume that (13.3) holds; rather, the most we can assume in general is that

$$0 \;\leq\; p_1, p_2 \;\leq\; p_1 + p_2 \;\leq\; 2. \tag{13.22}$$

Now, from Nation 1's perspective, p_2 also is the realized value of a random variable, say X, and so the point (X, Y) is continously distributed over the unit square $[0,1] \times [0,1]$. Let us assume that this distribution is uniform, so that the joint pdf of X and Y is 1.

Then because Nation 2 will desist from war when (13.8) holds and otherwise declare war, and because $V(x) \geq p_2 - c_2$ for Nation 2 (which knows p_2) translates to $V(x) \geq X - Y$ for Nation 1 (which knows neither p_2 nor c_2), the payoff to Nation 1 from unilaterally shifting the status quo to x is the random variable

$$F_1 \;=\; \begin{cases} p_1 - c_1 & \text{if } V(x) < X - Y \\ U(x) & \text{if } V(x) \geq X - Y \end{cases} \;=\; \begin{cases} x & \text{if } \Theta \leq 1 - x \\ p_1 - c_1 & \text{if } \Theta > 1 - x \end{cases} \tag{13.23}$$

by risk-neutrality, where

$$\Theta \;=\; X - Y \tag{13.24}$$

is distributed over $[-1, 1]$. So Nation 1's reward from choosing x is

$$\begin{aligned} f_1(x) \;&=\; E[F_1] \;=\; x \cdot \text{Prob}(\Theta \leq 1 - x) + (p_1 - c_1) \cdot \text{Prob}(\Theta > 1 - x) \\ &=\; x\, G_\Theta(1 - x) + \{p_1 - c_1\}\{1 - G_\Theta(1 - x)\} \\ &=\; p_1 - c_1 + \{x - p_1 + c_1\} G_\Theta(1 - x). \end{aligned} \tag{13.25}$$

Because (X, Y) is uniformly distributed over $[0, 1] \times [0, 1]$, for $\theta \in [-1, 1]$ we obtain

$$G_\Theta(\theta) \;=\; \text{Prob}(\Theta \leq \theta) \;=\; \text{Prob}(X - Y \leq \theta) \;=\; \begin{cases} \frac{1}{2}(1 + \theta)^2 & \text{if } -1 \leq \theta < 0 \\ 1 - \frac{1}{2}(1 - \theta)^2 & \text{if } 0 \leq \theta \leq 1. \end{cases} \tag{13.26}$$

So, for $1 - x \in [0, 1]$, (13.25) reduces to

$$\begin{aligned} f_1(x) \;&=\; p_1 - c_1 + \{x - p_1 + c_1\}\{1 - \tfrac{1}{2}x^2\} \\ &=\; x - \tfrac{1}{2}\{x - p_1 + c_1\}x^2, \end{aligned} \tag{13.27}$$

which is maximized by

$$x^* \;=\; \begin{cases} \frac{1}{3}\{p_1 - c_1 + \sqrt{(p_1 - c_1)^2 + 6}\} & \text{if } p_1 < \frac{1}{2} + c_1 \\ 1 & \text{if } p_1 \geq \frac{1}{2} + c_1 \end{cases} \tag{13.28}$$

with

$$f_1(x^*) \;=\; \begin{cases} \frac{1}{54}\Delta(\sqrt{\Delta^2 + 6} + \Delta)(\sqrt{\Delta^2 + 6} + \Delta + 12/\Delta) & \text{if } \Delta < \frac{1}{2} \\ \frac{1}{2}(1 + \Delta) & \text{if } \Delta \geq \frac{1}{2} \end{cases} \tag{13.29}$$

where

$$\Delta \;=\; p_1 - c_1. \tag{13.30}$$

The corresponding probability of war is now

$$\begin{aligned} P_w \;&=\; \text{Prob}(V(x^*) < \Theta) \;=\; \text{Prob}(\Theta > 1 - x^*) \\ &=\; 1 - G_\Theta(1 - x^*) \;=\; \tfrac{1}{2}x^{*2} \\ &=\; \begin{cases} \frac{1}{18}\{p_1 - c_1 + \sqrt{(p_1 - c_1)^2 + 6}\}^2 & \text{if } p_1 < \frac{1}{2} + c_1 \\ \frac{1}{2} & \text{if } p_1 \geq \frac{1}{2} + c_1 \end{cases} \end{aligned} \tag{13.31}$$

which again is always positive. In fact P_w invariably exceeds $\frac{1}{3}$, whereas p_w can be close to zero (when c_1 is small). Thus uncertainty over capabilities compounds uncertainy over costs, increasing the probability of war.

How much is the overall probability of war increased from the viewpoint of an external observer? Again, we can answer this question by assuming that (p_1, c_1) is uniformly distributed over the unit square. Then the overall probability of war can be assessed as

$$\begin{aligned} \mathrm{E}[P_w] &= \int_0^1 \int_0^{\min(\frac{1}{2}+c_1,1)} \tfrac{1}{18}\left\{p_1 - c_1 + \sqrt{(p_1-c_1)^2+6}\right\}^2 \cdot 1\, dp_1\, dc_1 + \int_0^{\frac{1}{2}} \int_{\frac{1}{2}+c_1}^{1} \tfrac{1}{2} \cdot 1\, dp_1\, dc_1 \\ &= \tfrac{719}{864} - \tfrac{4}{27}\sqrt{7} - \tfrac{1}{2}\operatorname{arccoth}(5) - \tfrac{1}{2}\operatorname{arccsch}(\sqrt{6}) + \tfrac{1}{2}\ln\left(\tfrac{3}{2}\right) \approx 0.3427, \qquad (13.32) \end{aligned}$$

which of course exceeds $\frac{1}{3}$. So, comparing with (13.21), we find that the overall probability of war increases by almost $\frac{2}{3}$.