Third Assignment

Due in ink at 1:25 p.m. on Monday, March 27, 2017

1. Find all admissible extremals for

$$J[y] \;=\; \int_0^b (x+1)y'^2\,dx$$

with $y(0) = 0$ and $b > 0$ when (b, β) must lie on $y = 1 + \ln(x+1)$. [10]

2. Find all admissible extremals for

$$J[y] \;=\; \int_0^1 \left\{y'^2 + yy' + y' + \tfrac{1}{2}y\right\} dx$$

when

(a) $y(0) = 0$ but $y(1) = \beta$ is free.

(b) $y(1) = 0$ but $y(0) = \alpha$ is free.

In each case, discuss whether a minimum is achieved. [10]

3. Find an admissible extremal for the problem of minimizing

$$J[y] \;=\; \int_0^1 \{y^2 + y'^2 + 2ye^{2x}\}\,dx$$

with $y(0) = \frac{1}{3}$, $y(1) = \frac{1}{3}e^2$. Show that it satisfies the sufficient condition, explicitly identifying both a suitable field of extremals and its associated direction field. [10]

4. Find an admissible extremal for the problem of minimizing

(a) $$J[y] \;=\; \int_0^2 y'^2\,dx \quad \text{subject to} \quad \int_0^2 y\,dx \;=\; 8$$

with $y(0) = 1$, $y(2) = 3$ and for the problem of minimizing

(b) $$J[y] \;=\; \int_1^3 y'^2\,dx \quad \text{subject to} \quad \int_1^3 y\,dx \;=\; 2$$

with $y(1) = 2$, $y(3) = 4$. [10]

5. For the problem of minimizing

$$J[y] \;=\; \int_0^2 \sqrt{1 + y^2y'^2}\,dx$$

with $y(0) = 1$ and $y(2) = 3$, obtain two fields of extremals containing the admissible extremal. Show that their direction fields satisfy (12.2) identically. [10]

[Perfect score: $5 \times 10 = 50$]