

Second Test

Tuesday, March 1, 2005

You are allowed to use a TI-30Xa (or any four-function calculator). No other calculator is allowed. You have 75 minutes. Present your solutions clearly. Show all necessary steps in your method. Include enough comments or diagrams to convince me that you thoroughly understand. Begin each question (as opposed to part of question) on a fresh sheet of paper, use *one* side of the paper only, and ensure that your solutions are stapled together in the proper order at the end of the test. You may assume that

$$\frac{d}{du} \{ \sinh^{-1}(u) \} = \frac{1}{\sqrt{u^2 + 1}}, \quad \frac{d}{du} \{ \cot(u) \} = -\frac{1}{\sin^2(u)}$$

(for $0 < u < \pi$ in the second case).

DO NOT WRITE ON THIS QUESTION PAPER, WHICH MUST BE TURNED IN AT THE END OF THE TEST (BUT NOT STAPLED TO YOUR SOLUTIONS)

1. What is the tangent line to the curve $y = \frac{(3x + 1)e^{x^2}}{x + 3}$ at the point $(1, e)$? [11]

2. Find both $m = \left. \frac{dy}{dx} \right|_{(x,y)=(-1,1)}$ and $\alpha = \left. \frac{d^2y}{dx^2} \right|_{(x,y)=(-1,1)}$ for $2x^3 + y^4 + 2 = y$ [11]

3. In each of the following cases, find $m = \left. \frac{dy}{dx} \right|_{x=0}$ *exactly*:
 - (a) $y = \ln(1 + e^{3x})$ [3]
 - (b) $y = \ln(\sqrt{x^2 + 6x + 3}) + \sinh^{-1}(5x)$ [4]

4. A particle moves along the x -axis with position $x = \frac{\ln(1 + t)}{1 + t}$ at time t .
 - (a) Find the particle's velocity at time t [3]
 - (b) Find the particle's acceleration at time t [3]
 - (c) Find the *precise* instant at which the velocity equals zero. [2]
 - (d) Find the *precise* instant at which the acceleration equals zero. [2]

5. A kite 30 m above the ground moves horizontally at a speed of 3 m/s. At what rate is the angle between the string and the horizontal decreasing when 60 m of string have been let out? [6]