1. $y = \left(\frac{3x+1}{x^2+3}\right)^{1/2} \Rightarrow \ln(y) = \frac{1}{2}\left\{\ln(3x+1) - \ln(x^2+3)\right\}$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left\{\frac{1}{3x+1}(3+0) - \frac{1}{x^2+3}\cdot(2x+0)\right\}$. Put $x = 1 \Rightarrow$

$y = \left(\frac{3+1}{1+3}\right)^{1/2} = 1$ and $\frac{1}{1}\cdot m = \frac{1}{2}\left\{\frac{3}{3+1} - \frac{2}{1+3}\right\}$ or $m = \frac{1}{8}$.

2. $\frac{dy}{dx} = (2\cdot 1+0)e^{x^2} + (2x+1)\cdot e^{x^2}\cdot 2x \Rightarrow m = \left.\frac{dy}{dx}\right|_{x=1} = 2e^{1^2} + 3e^{1^2}\cdot 2 = 8e$. So tangent line is

$y - 3e = m(x-1) \Rightarrow y - 3e = 8e(x-1)$ or $y = e(8x-5)$

3. $\frac{d}{dx}(x^2) + \frac{d}{dx}(y^3) + \frac{d}{dx}(y^4) = \frac{d}{dx}(1) \Rightarrow 2x + 3y^2\frac{dy}{dx} + 4y^3\frac{dy}{dx} = 0 \quad (*)$

$\Rightarrow 2\cdot 1 + 3(-1)^2 m + 4(-1)^3 m = 0 \Rightarrow m = 2$. Differentiating

$(*)$: $2 + 3\frac{d}{dx}\left(y^2\frac{dy}{dx}\right) + 4\frac{d}{dx}\left(y^3\frac{dy}{dx}\right) = 0 \Rightarrow 2 +$

$3\left\{2y\frac{dy}{dx}\frac{dy}{dx} + y^2\frac{d^2y}{dx^2}\right\} + 4\left\{3y^2\frac{dy}{dx}\frac{dy}{dx} + y^3\frac{d^2y}{dx^2}\right\} = 0 \Rightarrow$

$2 + 3\left\{-2m^2 + (-1)^2\alpha\right\} + 4\left\{3(-1)^2m^2 + (-1)^3\alpha\right\} = 0 \Rightarrow$

$\alpha = 2 - 6m^2 + 12m^2 = 2 + 6m^2 = 26$.

4 (a) $\ln(y) = 3\ln\left(\ln[1+(x+1)^4]\right) \Rightarrow \frac{1}{y}\frac{dy}{dx} = 3\cdot\frac{1}{\ln[1+(x+1)^4]}\frac{d}{dx}\left\{\ln[1+(x+1)^4]\right\}$

$= \frac{3}{\ln[1+(x+1)^4]}\cdot\frac{1}{1+(x+1)^4}\cdot\frac{d}{dx}\left\{1+(x+1)^4\right\} = \frac{3}{\ln[1+(x+1)^4]}\cdot\frac{1}{1+(x+1)^4}\left\{0+4(x+1)^3\right\}$

Now put $x = 0$. Get $\frac{1}{\ln(2)^3}\cdot m = \frac{3}{\ln(2)}\cdot\frac{1}{2}\cdot 4 \Rightarrow m = 6\{\ln(2)\}^2$.

(b) With $u = 3x$ and $v = 4x$ we have $\frac{dy}{dx} = 1\cdot\tanh^{-1}(3x) +$

$x\cdot\frac{1}{1-u^2}\frac{du}{dx} + \frac{1}{\sqrt{v^2+1}}\frac{dv}{dx} = \tanh^{-1}(3x) + \frac{3x}{1-(3x)^2} + \frac{4}{\sqrt{(4x)^2+1}}$

So $m = \tanh^{-1}(0) + 0 + \frac{4}{\sqrt{0+1}} = 0 + 0 + 4 = 4$

5 (a) $x = (2t-1)e^{-2t} \Rightarrow \frac{dx}{dt} = (2\cdot 1 - 0)e^{-2t} + (2t-1)\left\{e^{-2t}(-2)\right\}$

$= 2e^{-2t}\left\{1-(2t-1)\right\} = 4(1-t)e^{-2t}$

(b) $\frac{d^2x}{dt^2} = 4\frac{d}{dt}\left\{(1-t)e^{-2t}\right\} = 4\left\{(0-1)e^{-2t} + (1-t)e^{-2t}(-2)\right\}$

$= 4e^{-2t}\left\{-1 + 2(t-1)\right\} = 4(2t-3)e^{-2t}$

(c) Moves forward for $0 < t < 1$, decelerating; moves backward for $t > 1$, accelerating for $1 < t < 3/2$, decelerating for $t > 3/2$, and approaching $x = 0$ in the limit as $t \to 0$.

PHEW! MADE IT. YOU CAN SEE I WAS DETERMINED TO FIT THE WHOLE THING ON A SINGLE PAGE!