

## Third Test

Wednesday, July 14, 2004

You are allowed to use a TI-30Xa (or any four-function calculator). No other calculator is allowed. You have one hour. Present your solutions clearly. Show all necessary steps in your method. Include enough comments or diagrams to convince me that you thoroughly understand. Begin each question (as opposed to part of question) on a fresh sheet of paper, use *one* side of the paper only, and ensure that your solutions are stapled together in the proper order at the end of the test. You may assume that

$$\frac{d}{d\xi} \{\arctan(\xi)\} = \frac{1}{\xi^2 + 1}.$$

1. Given that  $f(1) = \frac{3}{10}$ ,  $f'(4) = -1$  and  $f''(t) = 3\sqrt{t}$  for all  $t > 0$ , find  $f(t)$  *exactly*. [8]

2. (a) Calculate  $\frac{d}{dx} \{\ln(\sqrt{x^2 + 1})\}$ . [2] (b) Find the *exact* value of  $\int_1^{\sqrt{3}} \frac{1 + 4x}{x^2 + 1} dx$ . [5]

**Hint:** Rewrite the integral as a sum of integrals you know how to calculate.

3. In each of the following cases, find the exact value of the definite integral:

(a)  $I = \int_1^4 \left\{ x - \frac{1}{\sqrt{x}} \right\}^2 dx$  [6]

(b)  $I = \int_0^{\pi/2} |\cos(2x)| dx$  [6]

(c)  $I = \int_{-1}^1 (3x + 4)(5x - 1) dx$  [6]

4. Show that the area of the region bounded above by the line  $y = 1$ , to the left by the line  $x = 0$  and below by the graph  $y = \sin(x)$  for  $0 \leq x \leq \frac{1}{2}\pi$  exceeds the area of the region bounded below by the line  $y = \frac{1}{2}$ , to the left by the line  $x = 0$  and above by the graph  $y = \frac{1}{x+1}$  for  $0 \leq x \leq 1$ . [8]

**Hint:** Find them both and compare!

5. A piece of wire 8 m long is cut into two pieces. One piece is bent into a circle and the other piece is bent into an equilateral triangle. What is the minimum total area that the two pieces of wire can enclose? [9]

**Hint:** Let each side of the equilateral triangle have length  $2x$ .