

Mock Third Test  
 Tuesday, October 26, 2004

You are allowed to use a TI-30Xa (or any four-function calculator). No other calculator is allowed. You have 75 minutes. Present your solutions clearly. Show all necessary steps in your method. Include enough comments or diagrams to convince me that you thoroughly understand. Begin each question (as opposed to part of question) on a fresh sheet of paper, use *one* side of the paper only, and ensure that your solutions are stapled together in the proper order at the end of the test.

DO NOT WRITE ON THIS QUESTION PAPER, WHICH MUST BE TURNED IN AT THE END OF THE TEST (BUT NOT STAPLED TO YOUR SOLUTIONS)

1. A function  $g$  is defined by

$$g(t) = \begin{cases} t^3 + 2t & \text{if } -\infty < t < 2 \\ t^2 - 9t^3 + 5t^4 & \text{if } 2 \leq t < \infty. \end{cases}$$

- (a) Why must  $g$  be continuous on  $(-\infty, \infty)$ ? [2]  
 (b) Find the *exact* value of  $\int_1^3 g(t) dt$ . [8]

2.  $F$  is defined on  $[0, 7]$  by  $F(t) = \int_0^t f(x) dx$  where

$$f(x) = \begin{cases} 3 + 2x & \text{if } 0 \leq x < 2 \\ 13 - 3x & \text{if } 2 \leq x < 4 \\ x - 3 & \text{if } 4 \leq x < 5 \\ 2 & \text{if } 5 \leq x \leq 7. \end{cases}$$

Find an explicit formula for  $F(t)$  for all  $t \in [0, 7]$ . [10]

3. Use the substitution  $u = \sqrt{2x + 3}$  to find the *exact* value of  $I = \int_{\frac{1}{2}}^3 \frac{7x + 6}{\{2x + 3\}^{7/2}} dx$ . [18]

**Note:** The correct answer exceeds  $\frac{1}{13}$  but is less than  $\frac{1}{12}$ .

4. In each of the following cases, find the exact value of the definite integral:

(a)  $I = \int_1^2 \left\{ x - \frac{2}{\sqrt{x}} \right\}^4 dx$  [6]

(b)  $I = \int_{-1}^1 (3 - 4x)(4 - 3x)(5 - 2x) dx$  [6]

(c)  $I = \int_0^1 |e^{2x} - 2| dx$  [6]

5. Given that  $f(1) = 8$ ,  $f'(1) = 48$  and

$$f''(t) = \frac{15(5t - 3)(5t^2 + 6t + 9)}{4t^3\sqrt{t}}$$

for all  $t > 0$ , find  $f(t)$  *exactly*. [12]

6. For  $b > 1$ , solve the equation  $\int_1^b \frac{1}{x^4} dx = \frac{7}{24}$ . [12]