1 Various remarks and comments at the beginning of the lecture regarding previous lectures

Remark 1.1. Restriction maps need not be injective.

Germs of sections of vector bundles form a sheaf (See Hartshorne Ex II.1.?)

Regular functions on a variety form a sheaf with the additional condition that $\mathcal{F}(\emptyset) = \{0\}$.

2 Discussion about topics for the courses over the next two semesters

Material omitted here. (It will probably appear in the syllabus when the next course is taught.)

3 Proj

Recall: If $A$is a ring, we define Spec$A$ which is the analog of an affine variety.

What is the analog of a projective variety in the theory of schemes?

(One answer is to use the functor t: varieties $\to$ schemes.)

Let S be a graded ring, i.e. there exists a decomposition

$$S = \bigoplus_{d \geq 0} S_d$$

as a direct sum of abelian groups such that $\forall d, e \geq 0$ $S_d S_e \subseteq S_{d+e}$.

Example 3.1. $S = k[x_0, \ldots, x_n]$ $S_d = k$ linear combinations of elements of S of degree d.

Definition 3.2. An element of S_d is said to be a *homogeneous element of S of degree d.*

An ideal $\mathfrak{a}$ of S is said to be homogeneous if $\mathfrak{a} = \bigoplus_{d \geq 0} (\mathfrak{a} \cap S_d)$ or equivalently if $\mathfrak{a}$ can be generated by homogeneous elements. ($\forall f \in \mathfrak{a}$, if $f = \sum f_d$ then $f_d \in \mathfrak{a}$.)

Example 3.3. $(x^2 + y^2) \subseteq k[x, y]$ is not homogeneous.

Example 3.4. $(x, y^2 + xy)$ is homogeneous.

Definition 3.5. S_+ denotes the ideal $\bigoplus_{d>0} S_d$ and is called the irrelevant ideal.

Example 3.6. In $k[x, y, z]$, $S_+ = (x, y, z)$.

Definition 3.7. The set Proj S consists of homogeneous prime ideals $\mathfrak{p}$ that do not contain S_+.