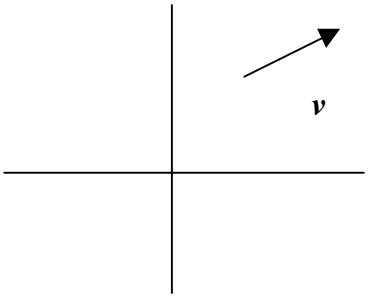
8.4
Vectors

A vector is a line segment with a direction. The length of the vector is called its magnitude.



Terminal point

Initial point

$||\boldsymbol{v}||$ is the magnitude of $\boldsymbol{v}$

$2\boldsymbol{v}$ means the vector in the same direction as $\boldsymbol{v}$ with magnitude $2||\boldsymbol{v}||$
2 is called a scalar

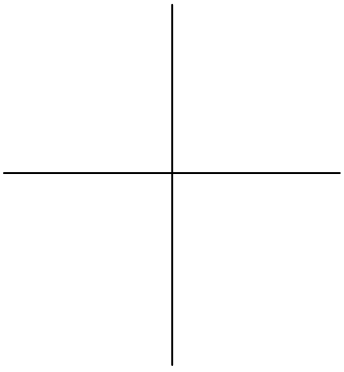
If a scalar is negative, the resulting vector has the opposite direction of $\boldsymbol{v}$.

Draw -2v on graph paper.

Vectors may be added together.
v + w is shown graphically:

Vectors with an initial point at (0,0) are called *position vectors*.

$\boldsymbol{v}$ = ai + bj components of $\boldsymbol{v}$ are a and b & (a, b) is the terminal point of v



i is a unit vector (its magnitude is 1) in the direction of the positive x-axis
j is a unit vector (its magnitude is 1) in the direction of the positive y-axis

Vector Operations

For $\boldsymbol{v} = a_1 i + b_1 j$ and $\boldsymbol{w} = a_2 i + b_2 j$:

$\boldsymbol{v} + \boldsymbol{w} = (a_1 + a_2)i + (b_1 + b_2)j$
$\boldsymbol{v} - \boldsymbol{w} = (a_1 - a_2)i + (b_1 - b_2)j$

and for scalar α:

$\alpha v = (\alpha a_1)i + (\alpha b_1)j$

$||\boldsymbol{v}|| = \sqrt{a_1^2 + b_1^2}$

EXAMPLES
For $v = 2i + 3j$ and $w = -4i + j$, find:

1) $2v - 3w$

2) $||\boldsymbol{v}||$

3) $||2v - 3w||$

Unit Vector in the direction of v

u = $\frac{v}{\|v\|}$ is the unit vector (magnitude = 1) in the same direction as v

EXAMPLE: If v = 3i – 2j, then what is the unit vector in the same direction as v?

EXAMPLE of eGrade question #154:

A) If P = (3,4) and Q = (5, 7), then PQ =

B) Now find QP.

Note: There are two types of multiplication with vectors. One is scalar multiplication, mentioned in this unit. The second is called a "dot product" of vectors. We will cover that one in 8.5. For now, understand that scalar multiplication produces a new vector, while adot product produces a real number.