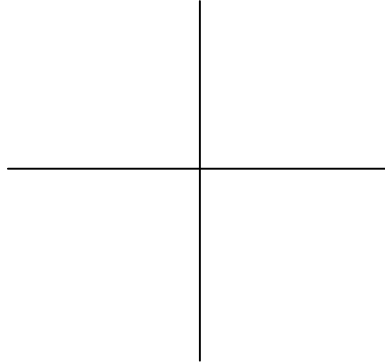**8.2**
**Polar Equations and Graphs**
eGrade questions 135-142

We will use some helpful identities:

Equal values: $\sin \theta = \sin (\pi - \theta)$ $\cos \theta = \cos (- \theta)$

Opposites: $\cos \theta$ and $\cos (\pi - \theta)$, $\cos \theta$ and $\cos (\pi + \theta)$, $\sin \theta$ and $\sin (\pi + \theta)$, $\sin \theta$ and $\sin (-\theta)$

Symmetry

1) Origin                             2) x-axis                             3) y-axis

Check for symmetry for:

1) $r = 4\sin \theta$                             2) $r = 6 + 4\cos \theta$

3) $r = 2 - 3\sin \theta$

4) $r = 2 \sin 2\theta$

5) $r = 2\cos 3\theta$

6) $r = 4\cos \theta$

# Polar Graphs

## Circles

a is a constant (a ≠ 0, but note that a may be positive or negative)

$r = a$            circle with radius | a | centered at the origin

$r = a \, cos \, \theta$     circle with center on the x-axis and tangent with respect to the origin

$r = a \, sin \, \theta$      circle with center on the y-axis and tangent with respect to the origin

## Cardiod (looks like a heart)

$r = a \pm a \, cos \, \theta$ symmetric with respect to x-axis

$r = a \pm a \, sin \, \theta$ symmetric with respect to y-axis

Examples:     $r = 2 - 2 \, cos \, \theta$

               $r = sin \, \theta - 1$

## Limaçon    (without inner circle)

$| \, a \, | > | \, b \, | \neq 0$

$r = a \pm b \, cos \, \theta$ symmetric with respect to x-axis

$r = a \pm b \, sin \, \theta$ symmetric with respect to y-axis

Examples:     $r = 3 + 2 \, cos \, \theta$

               $r = sin \, \theta - 2$

## Limaçon    (with inner circle)

$| b \, | > | a \, | \neq 0$

$r = a \pm b \, cos \, \theta$ symmetric with respect to x-axis

$r = a \pm b \, sin \, \theta$ symmetric with respect to y-axis

Examples:     $r = 3 \, cos \, \theta - 2$

               $r = 1 + 2sin \, \theta$

## 3 Petal Roses

$r = a \, cos \, 3\theta$     symmetric with respect to x-axis

$r = a \, sin \, 3\theta$      symmetric with respect to y-axis

Examples:     $r = -2 \, cos \, 3\theta$

               $r = \, sin \, 3\theta$

## 4 Petal Roses

$r = a \, cos \, 2\theta$     all types of symmetry – max distance from origin along axes

$r = a \, sin \, 2\theta$     all types of symmetry – max distance from origin along 45° lines

Examples

$r = -2 \, cos \, 2\theta$

$r = \, sin \, 2\theta$

Use only $\theta \geq 0$ when you try to match points.

## Spirals

$r = a\theta$

Graph:

1) r = θ

| θ | 0 | π/6 | π/4 | π/3 | π/2 |
|---|---|-----|-----|-----|-----|
| r |   |     |     |     |     |

2) r = −θ

| θ | 0 | π/6 | π/4 | π/3 | π/2 |
|---|---|-----|-----|-----|-----|
| r |   |     |     |     |     |

Other graphs:

1) rcos θ = -2                    2) θ = π/4