8.1
Polar Coordinates and Polar Equations

You may want to bring in a piece of polar graph paper. I have provided one copy for each student. You should make copies of it for yourself if you want to use it for homework or for section 8.2.

Polar Coordinates are (radius, angle) or (r, θ).

When graphing points in the polar plane:
1) Rotate from 0 to θ.
2) Go out r units along the terminal side of θ if $r > 0$ or go out r units along the ray opposite the terminal side of θ if $r < 0$. If $r = 0$ the point will be at the center of the graph no matter what the value of θ.

<u>Examples</u> Plot these points:

    1) $(3, \pi/6)$
    2) $(2, 5\pi/4)$
    3) $(-2, \pi/4)$
    4) $(-2, 9\pi/4)$
    5) $(3, 0)$
    6) $(-2, \pi/6)$

NOTE that only points with angles whose terminal sides coincide or angles that are half way around the circle from each other could be matching positions. See #2-4 above.

eGrade questions (8.1 includes questions 129 – 134)

    1) Select the alternate polar representation of the point with polar coordinates
       $(r, \theta) = (1, -3\pi/4)$
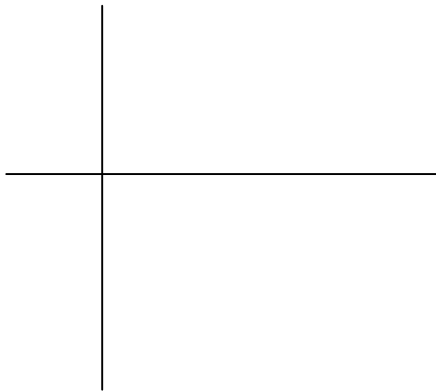   A) $(r, \theta) = (1, \pi/4)$
           B) $(r, \theta) = (-1, \pi/4)$
           C) $(r, \theta) = (1, 3\pi/4)$
           D) $(r, \theta) = (-1, 3\pi/4)$
           E) $(r, \theta) = (-1, 7\pi/4)$

2) Select ALL polar representations of the point with polar coordinates $(r, \theta) = (-3, 4\pi/3)$
           A) $(r, \theta) = (3, -5\pi/3)$
           B) $(r, \theta) = (-3, -2\pi/3)$
           C) $(r, \theta) = (-3, \pi/3)$
           D) $(r, \theta) = (3, -2\pi/3)$
           E) $(r, \theta) = (-3, -4\pi/3)$

NOTE: $(r, \theta) = (r, \theta + 2n\pi)$ for any integer n *and* $(r, \theta) = (-r, \theta + n\pi)$ for any odd integer n

Converting from polar coordinates to rectangular coordinate & vice versa.

P (r, θ)

P (x, y)

$$x^2 + y^2 = r^2$$

cos θ = $\Longrightarrow$ x =

sin θ = $\Longrightarrow$ y =

And in the 1st quadrant tan θ = $\Longrightarrow$ θ =

Let θ′ be the reference angle for θ.

Then, tan θ′ =

Examples:

1) Select the rectangular coordinates for the point with polar coordinates
   (r, θ) = (2, -3π/4)
   A) (x, y) = $(\sqrt{2}, -\sqrt{2})$ C) (x, y) = $(-\sqrt{2}, \sqrt{2})$
   B) (x, y) = $(-\sqrt{2}, -\sqrt{2})$ D) (x, y) = $(\sqrt{2}, \sqrt{2})$

2) Select the polar coordinates for the point with rectangular coordinates
   (x, y) = (-2, -2).

   A) (r, θ) = $(-2\sqrt{2}, -3π/4)$ C) (r, θ) = $(2\sqrt{2}, 5π/4)$
   B) (r, θ) = $(2\sqrt{2}, 7π/4)$ D) (r, θ) = $(-2\sqrt{2}, -π/4)$

Convert the polar equation to a rectangular equation.

1) $r = \dfrac{3}{1 - \cos\theta}$

2) $r = 5\sec\theta$

3) $r = -4\sin\theta$ (Hint: multiply the eq. by r)

4) $r^2 = \dfrac{4}{\sin\theta\cos\theta}$

5) $r(4\sin\theta - 2\cos\theta) = -3$

6) $r = 4$

Convert the polar equation to a rectangular equation.

3) $x^2 - y^2 = 9$

4) $2xy = 5$

5) $4x^2 + 4y^2 = 3$