6.5 Notes

Recall from algebra or pre-calculus how an inverse function is formed from a function.

y = f(x) contains points (x, f(x)) or (x, y)

The inverse of f, called f^{-1} will contain the same set of points but the coordinates are reversed (y, x).

For example, consider y = 3x or f(x) = 3x.

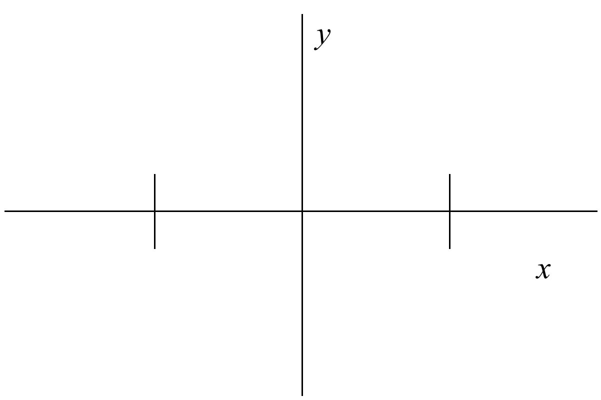
Here are some points belonging to this function: (-1, -3), (0,0), and (2, 6).

The inverse of this function will contain the points (-3, -1), (0,0), and (6,2). The formula for the inverse is y = (1/3)x or $f^{-1}(x) = (1/3)x$.

An important point to remember is that the only functions with inverses are those that are 1-1. Let us recall what that means:

Now let us consider the function y = sin x. Is the function 1-1?

It is useful in trigonometry to form an inverse function by limiting the domain of y = sin x so that an inverse function may be formed.



Let's write down some points on the function. They are all (x, sin x) or in words (angle value, sine of that angle).

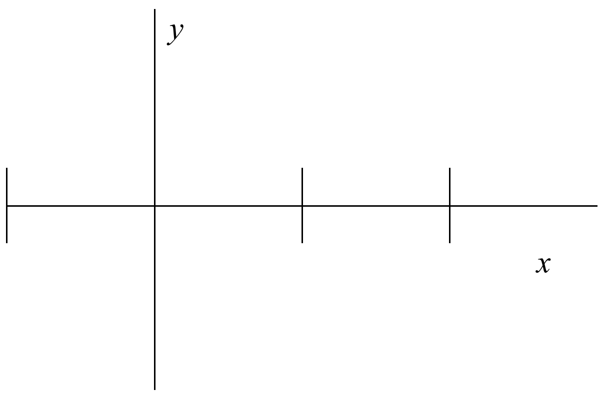
What would the corresponding inverse function points be?

Note that these have the form (sine of the angle, angle). The proper way to write this inverse function is $y = \sin^{-1} x$ or $y = \arcsin x$.

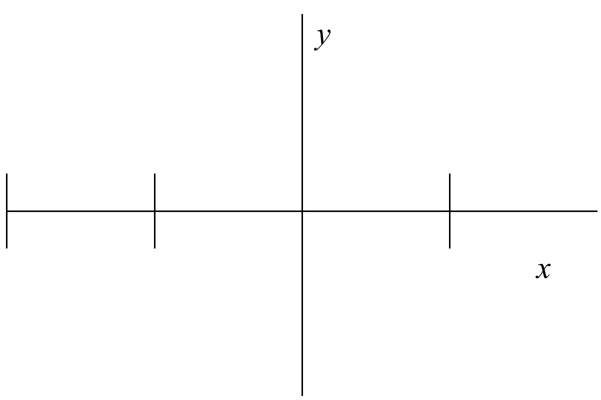
The domain we are using for $y = \sin x$ is $[-\pi/2, \pi/2]$ and the range is $[-1, 1]$. So the domain for $y = \sin^{-1} x$ is:

and the range is:

In a similar fashion we can form inverse functions for $y = \cos x$ and $y = \tan x$.



For $y = \cos^{-1} x$ or $y = \text{arcos } x$ we have domain ______________and range __________.



For $y = \tan^{-1} x$ or $y = \arctan x$ we have domain ______________and range __________.

YOU MUST MEMORIZE THE DOMAIN AND RANGE FOR EACH INVERSE FUNCTION!

We will be evaluating inverse trig functions for particular x values. It is helpful to note a few things.

$\sin^{-1} x$ or arcsin x is AN ANGLE. In particular, it is the angle whose sine value = x with the angle $\in [-\pi/2, \pi/2]$.
So if x is positive, then $\sin^{-1} x$ or arsin x is in quadrant ______.
And if x is negative, then $\sin^{-1} x$ or arcsin x is in quadrant ______.

$\cos^{-1} x$ or arccos x is AN ANGLE. In particular, it is the angle whose cosine value = x with
the angle $\in [-\pi/2, \pi/2]$.
So if x is positive, then $\cos^{-1} x$ or arccos x is in quadrant ______.
And if x is negative, then $\cos^{-1} x$ or arccos x is in quadrant ______.

$\tan^{-1} x$ or arctan x is AN ANGLE. In particular, it is the angle whose tangent value = x with
the angle $\in [-\pi/2, \pi/2]$.
So if x is positive, then $\tan^{-1} x$ or arctan x is in quadrant ______.
And if x is negative, then $\tan^{-1} x$ or arctan x is in quadrant ______.

Determine in which quadrant each angle lies (eGrade problem #78):

1) arcos (-0.5)

2) arcos (1/4)

3) arcsin (1/2)

4) arcsin (-1/3)

5) arctan (-2/3)

6) arctan (100π)

The 6.5 eGrade problems in mac1114 begin with #72.

1) Find $\sin^{-1}(-2\pi)$.

2) Find $\sin^{-1}(-1/2)$.

3) Find $\cos^{-1}(3)$.

4) Find $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$.

5) Find $\tan^{-1}(\sqrt{3})$.

6) Find $\tan^{-1}(-1)$.

Find each of the following (note: we need to distinguish between “none of these” and “not defined” as both may be given as eGrade answer choices)

1) $\cos(\sin^{-1}(-1.56))$

2) $\sin(\cos^{-1}(-1.56))$

3) $\tan(\cos^{-1}(0))$

4) $\tan(\sin^{-1}(-\frac{\sqrt{2}}{2}))$

5) $\cos(\sin^{-1}(1/3))$

6) $\sin(\tan^{-1}(-1/2))$

Problems with variables

If $1 < a$, then $\tan(\tan^{-1}(1/a)) =$

If $a > 0$, then $\sin(\tan^{-1} - \frac{\sqrt{a}}{2})$

If $a<1$, then $\sec(\sin^{-1} \frac{1}{\sqrt{a}}) =$

In algebra & pre-calculus we look at inverse functions for which $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$. Recall that $f(g(x))$ is called a composition of functions.

We have to be more careful with the composition of inverse trig functions.

1) $\sin(\sin^{-1}(x)) = x$ as long as $x \in [-1,1]$, otherwise it is not defined
2) $\cos(\cos^{-1}(x)) = x$ as long as $x \in [-1,1]$, otherwise it is not defined
3) $\tan(\tan^{-1}(x)) = x$ for all values of x since the domain of $y = \tan^{-1}(x)$

EXAMPLES:

1) a) $\sin(\sin^{-1}(1/2)) = 1/2$ b) $\sin(\sin^{-1}(2))$ is not defined

2) a) $\cos(\cos^{-1}(0.7)) = 0.7$ b) $\cos(\cos^{-1}(-\pi))$ is not defined

3) $\tan(\tan^{-1}(4\pi)) = 4\pi$

AND

1) $\sin^{-1}(\sin(x)) = x$ for all values x in the RANGE of $y = \sin^{-1}(x)$, otherwise you will need to find $\sin(x)$ and then find the inverse sine of that answer
2) $\cos^{-1}(\cos(x)) = x$ for all values x in the RANGE of $y = \cos^{-1}(x)$, otherwise you will need to find $\cos(x)$ and then find the inverse cosine of that answer
3) $\tan^{-1}(\tan(x)) = x$ for all values x in the RANGE of $y = \tan^{-1}(x)$, otherwise you will need to find $\tan(x)$ and then find the inverse tangent of that answer

EXAMPLES

1) a) $\sin^{-1}(\sin(\pi/2)) = \pi/2$ b) $\sin^{-1}(\sin(\pi)) = 0$

2) a) $\cos^{-1}(\cos(2\pi/3)) = 2\pi/3$ b) $\cos^{-1}(\cos(-2\pi/3)) = 2\pi/3$

3) a) $\tan^{-1}(\tan(-\pi/4)) = -\pi/4$ b) $\tan^{-1}(\tan(-3\pi/4)) = \pi/4$

FIND: $\tan^{-1}(\tan(23\pi/12)$