

Section 6.1 Verifying Identities

In higher math courses, including calculus, it is sometimes necessary to rewrite trigonometric expressions in different forms. In order to do this you may:

- 1) use known identities
- 2) perform algebraic operations
- 3) use acceptable algebraic manipulations such as “multiplying by the conjugate” or combining fractions

Recall these identities that may be used:

Reciprocal identities

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

Cofunction identities

If A and B are complimentary:

$$\sin A = \cos B$$

$$\tan A = \cot B$$

$$\sec A = \csc B$$

Written another way:

$$\sin \theta = \cos (\pi/2 - \theta)$$

$$\tan \theta = \cot (\pi/2 - \theta)$$

$$\sec \theta = \csc (\pi/2 - \theta)$$

$$\cos \theta = \sin (\pi/2 - \theta)$$

$$\cot \theta = \tan (\pi/2 - \theta)$$

$$\csc \theta = \sec (\pi/2 - \theta)$$

Pythagorean Identities

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

Even/Odd Identities

$$\cos(-\theta) = \cos \theta$$

$$\sin(-\theta) = -\sin \theta$$

$$\tan(-\theta) = -\tan \theta$$

$$\sec(-\theta) = \sec \theta$$

$$\csc(-\theta) = -\csc \theta$$

$$\cot(-\theta) = -\cot \theta$$

Useful Algebra formula: $(A - B)(A + B) = A^2 - B^2$

$$\frac{\cos \theta}{\sin \theta} = \cot \theta$$

ALSO USEFUL:

$$\frac{\sin \theta}{\cos \theta} = \tan \theta$$

EXAMPLES – For each example show the left side is equal to the right side.

1) $(1 + \sec \theta)(1 - \sec \theta) = -\tan^2 \theta.$

$$2) 1 - \frac{\cos^2 \theta}{1 - \sin \theta} = -\sin \theta$$

$$3) \csc \theta \cdot \cos \theta = \cot \theta$$

$$4) \frac{\sec \theta}{\csc \theta} + \frac{\sin \theta}{\cos \theta} = 2 \tan \theta$$

$$5) \frac{1 - \sin \theta}{1 + \sin \theta} = (\sec \theta - \tan \theta)^2$$

$$6) \frac{\tan \theta - \cot \theta}{\tan \theta + \cot \theta} = \sin^2 \theta - \cos^2 \theta$$

$$7) 1 - \frac{\sin^2(-\theta)}{1 + \cos(-\theta)} = \cos \theta$$

EGRADE PROBLEMS

8) Select ALL the equations which are identities

A) none of these are identities

B) $\cot^2 \theta = \csc^2 \theta - 1$

C) $\frac{\sin(-\theta)}{\tan \theta} = -\cos \theta$

D) $\frac{\cos \theta}{\cos \theta - 1} = -1$

9) Select ALL the equations which are NOT identities

A) all of these are identities

B) $\frac{\cot \theta - 1}{\cot \theta} = -1$

C) $\sec(-\theta) \cdot \cot \theta = -\csc \theta$

D) $\cos^2 \theta = 1 - \sin^2 \theta$