# Section 5.3

Periodic functions

Look at the unit circle and note that

$\sin(\theta \pm 2\pi) = \sin\theta$, in fact $\sin(\theta \pm 2n\pi)$, for all integer values of n
$\cos(\theta \pm 2\pi) = \cos\theta$, in fact $\cos(\theta \pm 2n\pi)$, for all integer values of n

AND

$\tan(\theta \pm \pi) = \tan\theta$, in fact $\tan(\theta \pm n\pi)$, for all integer values of n

The same is true for the reciprocal functions.

EXAMPLE 1: Use one of the facts above to find the exact value of sin 405°.

EVEN AND ODD FUNCTIONS

Definition of an even function: A function f is even if $f(-x) = f(x)$ for all values of x.

Which trigonometric functions are even?

NOTE: Even functions are symmetric to the y-axis (see $y = x^2$, for example).

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Definition of an odd function: A function f is odd if $f(-x) = -f(x)$ for all values of x.

Which trigonometric functions are odd?

NOTE: Even functions are symmetric to the origin (see $y = x^3$, for example).

EXAMPLE 2: If $\cot\theta = -3$, then $\cot\theta + \cot(\pi-\theta) + \cot(2\pi - \theta)$

EXAMPLE 3: Find the quadrant containing the terminal side of the angle $\theta$.

A) $\theta = 16\pi/3$

B) $\theta = -2\pi/3$

EXAMPLE 4: Find $\csc(-\pi/6) + \cot(-6\pi)$.

EXAMPLE 5: If $\csc \theta < 0$ and $\sec \theta < 0$, then $\theta$ lies in which quadrant?

EXAMPLE 6: If $\cos \theta = -\frac{3}{4}$ and $\sin \theta = \frac{\sqrt{7}}{4}$, then the exact value of $\cot \theta$ = ?

EXAMPLE 7: If $\cos \theta = -\frac{3}{5}$ and $\theta$ lies in quadrant III, then the exact value of $\tan \theta$ is?

EXAMPLE 8: If $\cot \theta = -3$ and $\theta$ lies in quadrant II, then the exact value of $\csc \theta$ is?

GRAPHS OF TRIGONOMETRIC FUNCTIONS

| Function | Domain | Range |
|---|---|---|
| y = sin x | $(-\infty,\infty)$ | |
| y = csc x | | |
| y = cos x | $(-\infty,\infty)$ | |
| y = sec x | | |
| y = tan x | | $(-\infty,\infty)$ |
| y = cot x | | $(-\infty,\infty)$ |

A HANDOUT WITH THE BASIC TRIG FUNCTION GRAPHS WILL BE PROVIDED IN CLASS.

NOTATION

$\sin^2\theta = (\sin\theta)^2$ $\cos^2\theta = (\cos\theta)^2$ $\tan^2\theta = (\tan\theta)^2$

PYTHAGOREAN IDENTITIES

$\cos^2\theta + \sin^2\theta = 1$

Dividing by $\sin^2\theta$ we get:

Dividing by $\cos^2\theta$ we get: