Vector Calculus Assignments

Problem numbers refer to the Schey textbook, 4th edition.

August 30: 1.1(a, b, d, f), 1.2–1.6, 2.1, 2.2
September 6: 2.4–2.8
September 13: 2.14, 2.16, 2.18, 2.22
September 20: 2.23, 2.24, 2.26
October 4:

1. Evaluate the line integral $\int_C y^3\,dx + (x^3 + 3xy^2)\,dy$ counterclockwise around the circle centered at the origin with radius 3. What is the value if integrated clockwise around the circle?

2. Evaluate the line integral $\int_C y\,dx + x^2\,dy$ where C is the parabolic arc given by $y = 4x - x^2$ from $(4, 0)$ to $(1, 3)$.

3. Let $\rho(x, y, z) = \frac{1}{2}(x^2 + y^2 + z^2)$ be the density of a spring. The spring forms a helix, described by the parameterized position vector $r(s) = 3\cos s\,\hat{\imath} + 3\sin s\,\hat{\jmath} + 2s\,\hat{k}$. Find the total mass of two turns of the spring.

4. Determine the work done by the force $\vec{F} = xy\hat{\imath} + z^2\hat{\jmath} + x\hat{k}$ in moving an object from location $(1, 0, 0)$ to $(4, 0, 9)$ along the curve given by $x = 1 + s$, $y = 0$, $z = s^2$, with $0 \le s \le 3$.

5. Determine the work done by the force $\vec{F} = y^2\hat{\imath} + x\hat{\jmath} + z\hat{k}$ in moving an object along the curve $z = y = e^x$ from $x = 0$ to $x = 1$.

6. Determine the work done by the force $\vec{F} = 5z^2\hat{\imath} + 2x\hat{\jmath} + (x + 2y)\hat{k}$ in moving an object along the curve $x = s$, $y = s^2$, $z = s^2$ with $0 \le s \le 1$.

Then repeat, but this time integrate along the straight line joining points $(0,0,0)$ and $(1,1,1)$. Are they the same?

7. Determine the work done by the force $\vec{F} = x\,\hat{\imath} + y\,\hat{\jmath} - 5z\,\hat{k}$ over the curve described by the parameterized position vector $\vec{r} = 2\cos s\,\hat{\imath} + 2\sin s\,\hat{\jmath} + s\,\hat{k}$ for $0 \le s \le 2\pi$.

October 11: 3.1, 3.3, 3.4ab, 3.6, 3.7

1. By calculating the line integral of $\vec{F} = y\,\hat{\imath} + (x^2 - x)\,\hat{\jmath}$ around the square in the xy-plane connecting the four points $(0,0)$, $(1,0)$, $(1,1)$, and $(0,1)$, show that $\vec{F}$ cannot be a conservative vector field.

October 18: 3.12–3.15
October 25: 3.18, 3.19, 3.26

1. Green's Theorem states that if $\vec{F} = f(x,y)\,\hat{\imath} + g(x,y)\,\hat{\jmath}$ and C is a closed, counterclockwise-oriented curve in the xy-plane, then

$$\oint_C f\,dx + g\,dy = \iint_S \frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\,dxdy$$

where S is the region in the xy-plane enclosed by C. Use this theorem to show that the area bounded by a closed curve C is given by

$$\frac{1}{2}\oint_C x\,dy - y\,dx \quad .$$

November 8: 4.1, 4.2(a,c,e,g), 4.3, 4.4, 4.13
November 15: 4.19, 4.20

1. Find the angle between the sphere $x^2 + y^2 + z^2 = 2$ and the cylinder $x^2 + y^2 = 1$ at a point of intersection.

2. Find the unit normal vector to the surface $xy^2 + 2yz = 4$ at the point (-2,2,3).